Dissipation of Crack Energy and Dynamic Mechanical Analysis

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Classical Fracture Mechanics

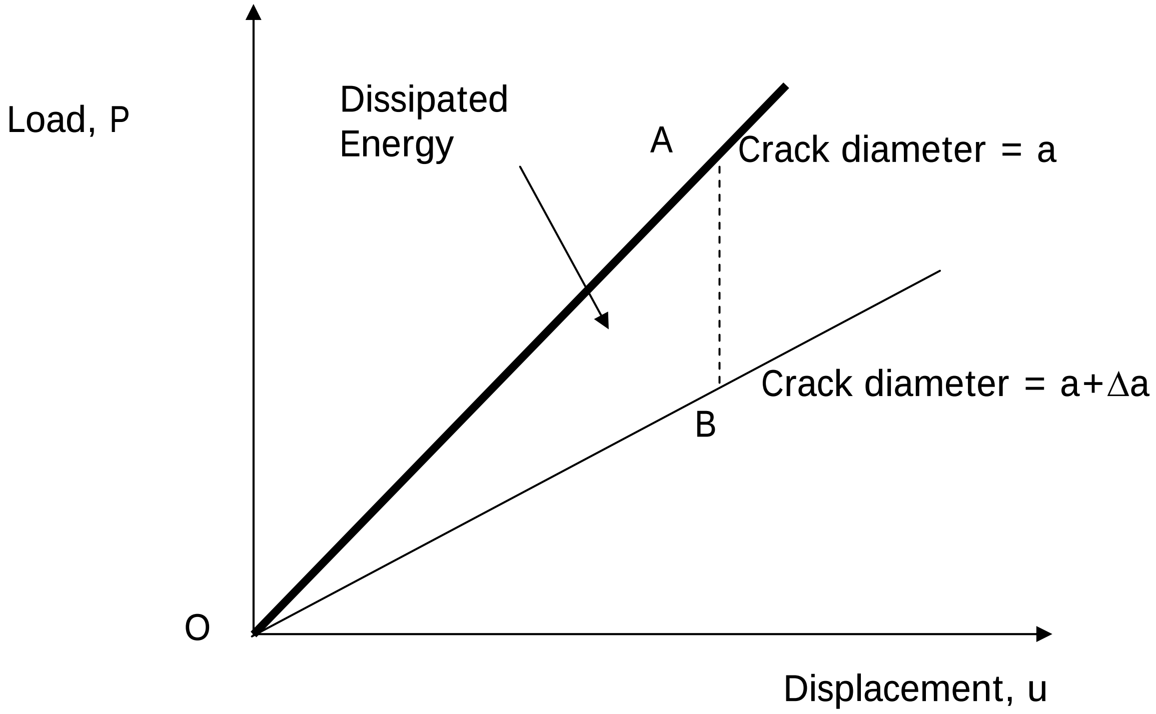
- Griffith's Theory

$$\frac{\partial W}{\partial A} - \frac{\partial U^e}{\partial A} = \frac{\partial \Gamma}{\partial A} = 2\gamma$$

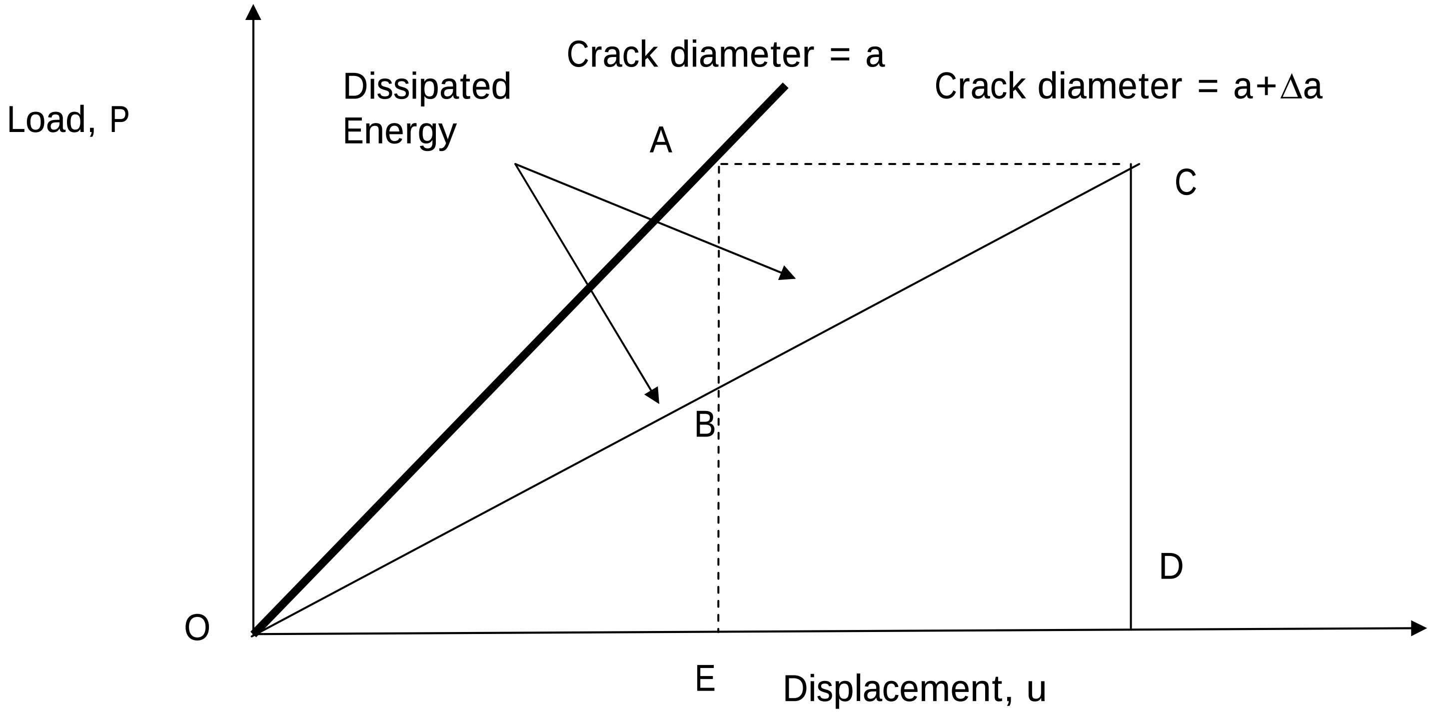
  - where W is the work performed by the applied loads during cracking,
  - U_e represents the elastic strain energy,
  - Γ is the energy spent in increasing the crack area,
  - γ is the surface energy.
  - The change in work performed by the applied loads is equal to zero under strain controlled loading. Therefore, the energy spent in increasing the crack comes from the release of elastic energy.

Controlled Strain

- Release of Elastic Energy = OAB



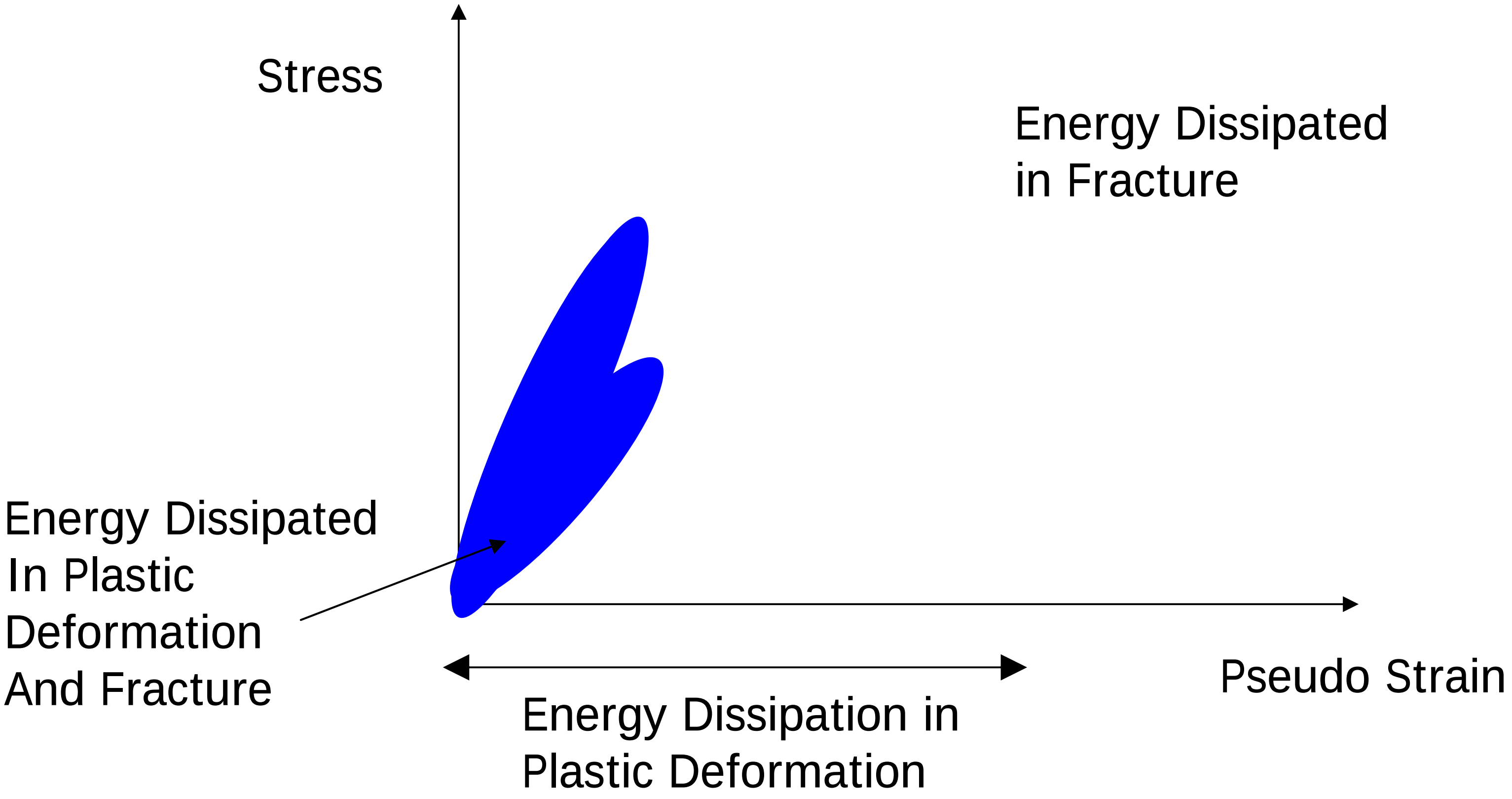
Controlled Stress



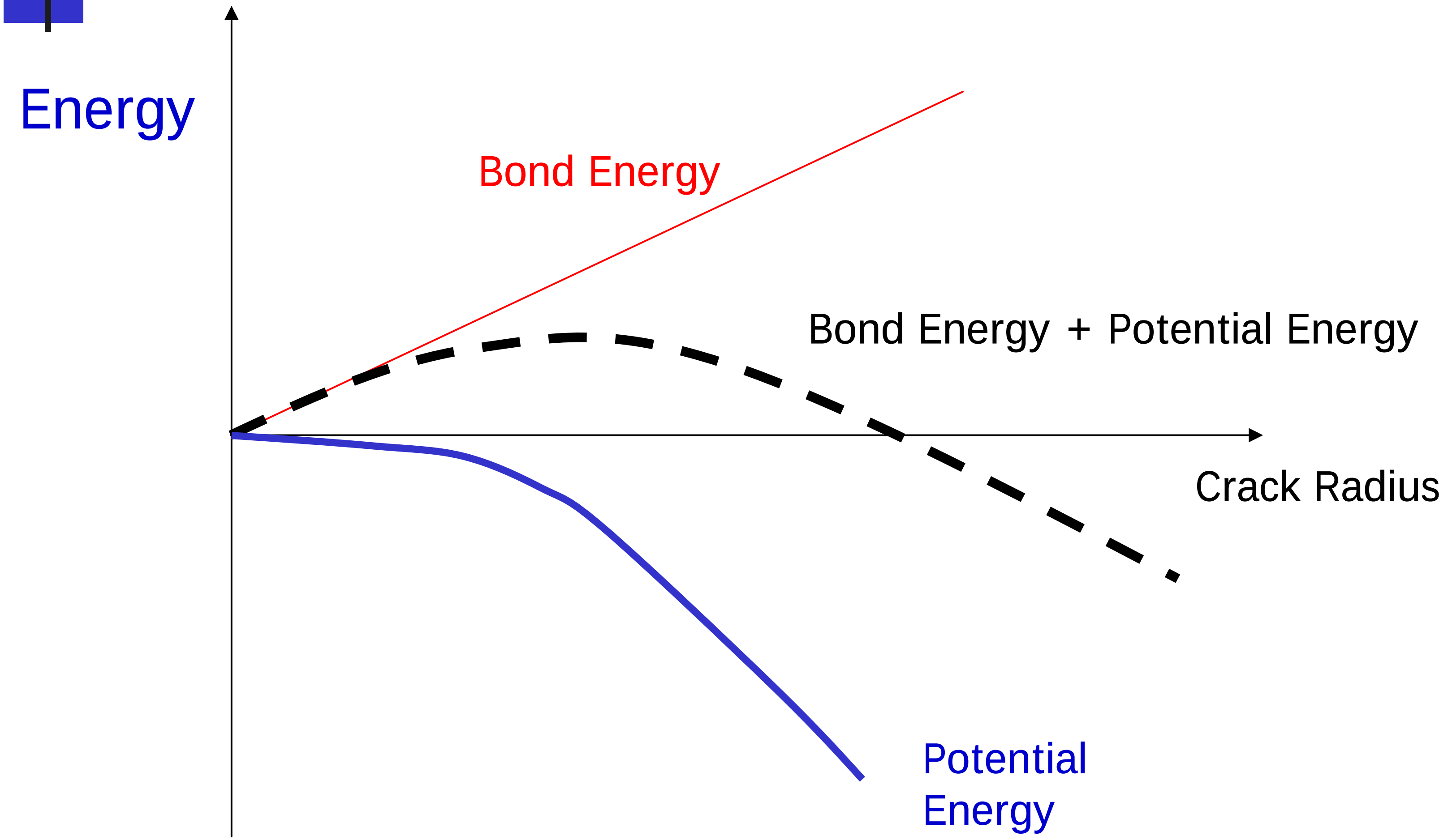
- For a stress controlled test, some energy is supplied by the external forces as the material experiences deformation at the onset of cracking.
- ACDE = $\frac{\partial W}{\partial A}$
- OCD - OAE = $\frac{\partial U^e}{\partial A}$
- According to Griffith's theory: The energy spent in increasing the crack is equal to the area OAC.

Controlled Stress vs. Controlled Strain

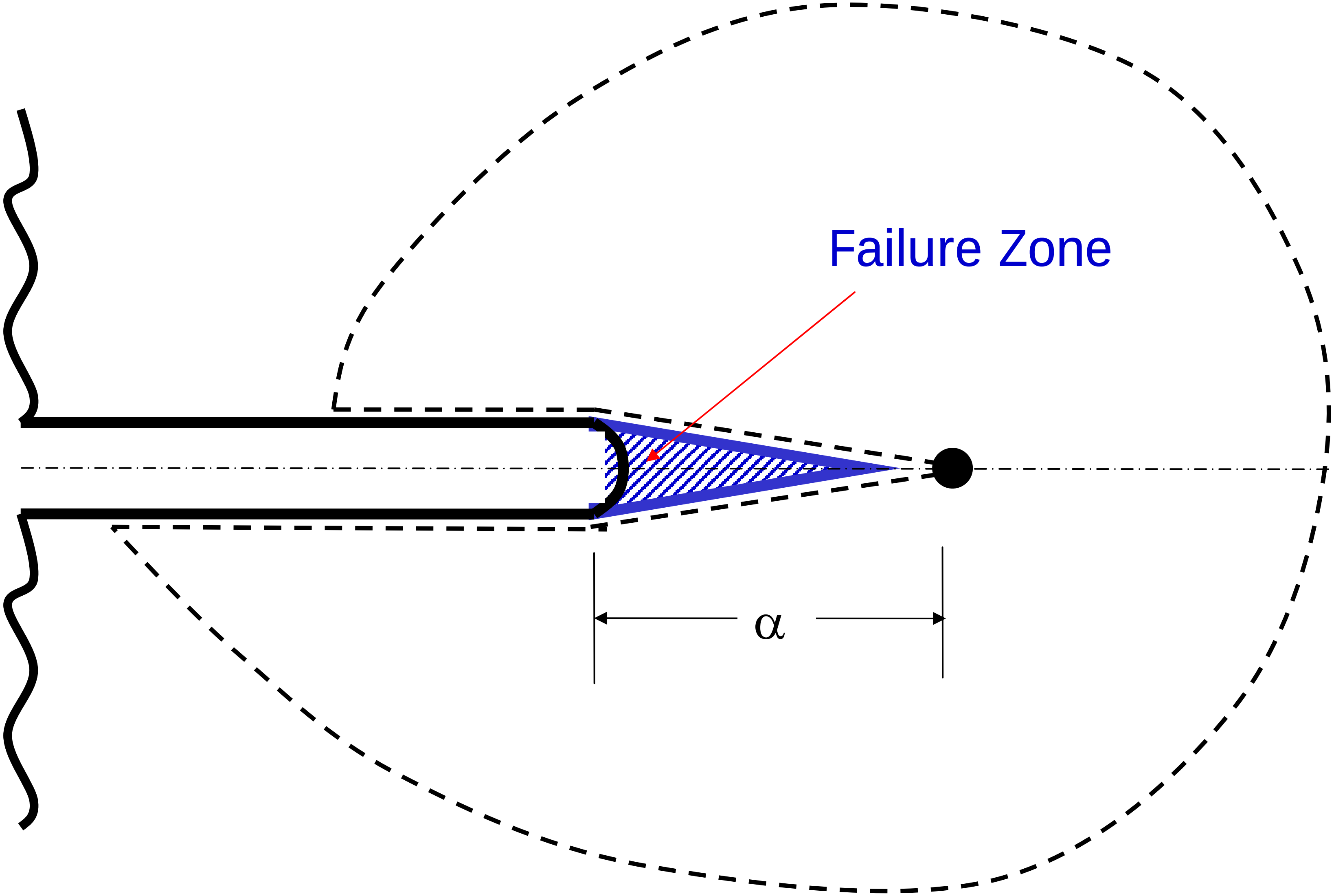
Controlled Strain



There is difference between stress controlled and strain controlled loadings in amount of energy dissipated in cracking and sources of this energy.


Energy
Bond Energy
Bond Energy + Potential Energy
Crack Radius
Potential Energy

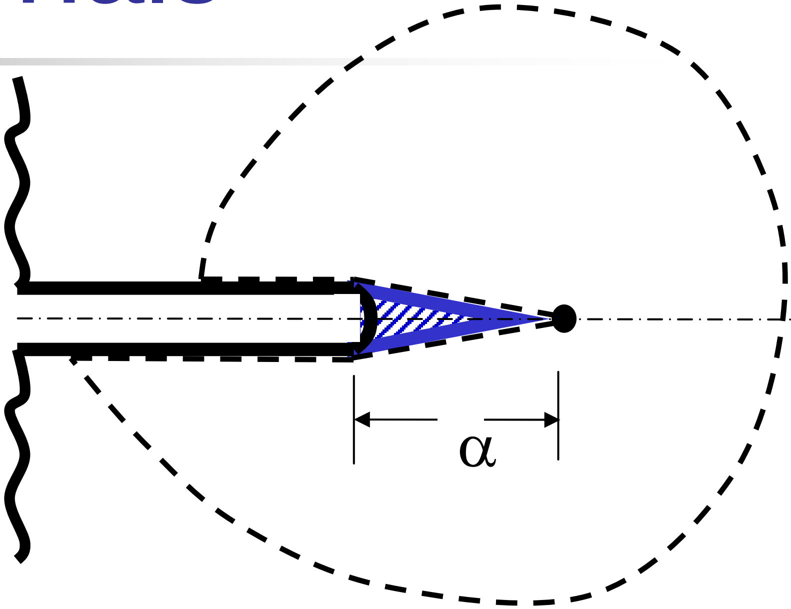
$$\frac{\partial W}{\partial A} - \frac{\partial U^e}{\partial A} = 2\left(\gamma + \gamma_p\right)$$



Failure Zone

Paris Fracture Law for Viscoelastic Materials

$$\frac{d\bar{r}}{dN} = A\left[J_R\right]^n$$



n = 1+1/m Tensile strength is constant

n = 1/m Bond energy and α are constants

m = is the exponent in the relaxation modulus equation

$$E(t) = E_\infty + E_1 t^{-m}$$

α(N) increases for stress controlled and decreases or is constant for strain controlled

Relationship Between Crack Size and DPSE (W_R)

$$\frac{d\bar{r}}{dN} = A\left[J_R\right]^n$$

$$J_R = \frac{\dfrac{\partial W_R}{\partial N}}{\dfrac{\partial(c.s.a)}{\partial N}}$$

$$\frac{\partial(c.s.a)}{\partial N} = 4\pi m\bar{r}\,\frac{\partial\bar{r}}{\partial N}$$

J_R = Pseudo J - Integral representing the amount of dissipated pseudo strain energy per unit area of crack surface area.

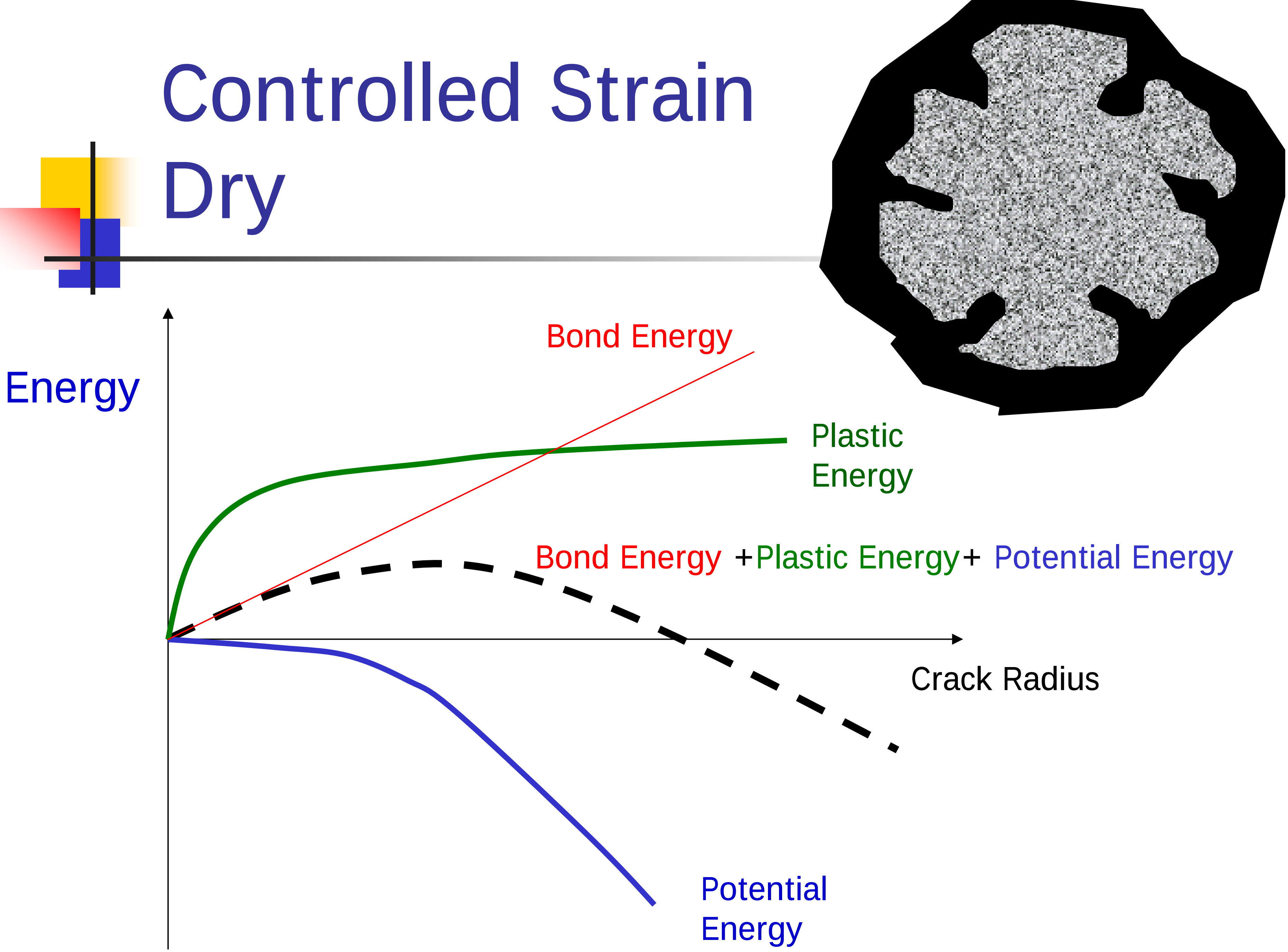
$c.s.a$ = Crack surface area.

$\bar{r}$ = Crack radius.

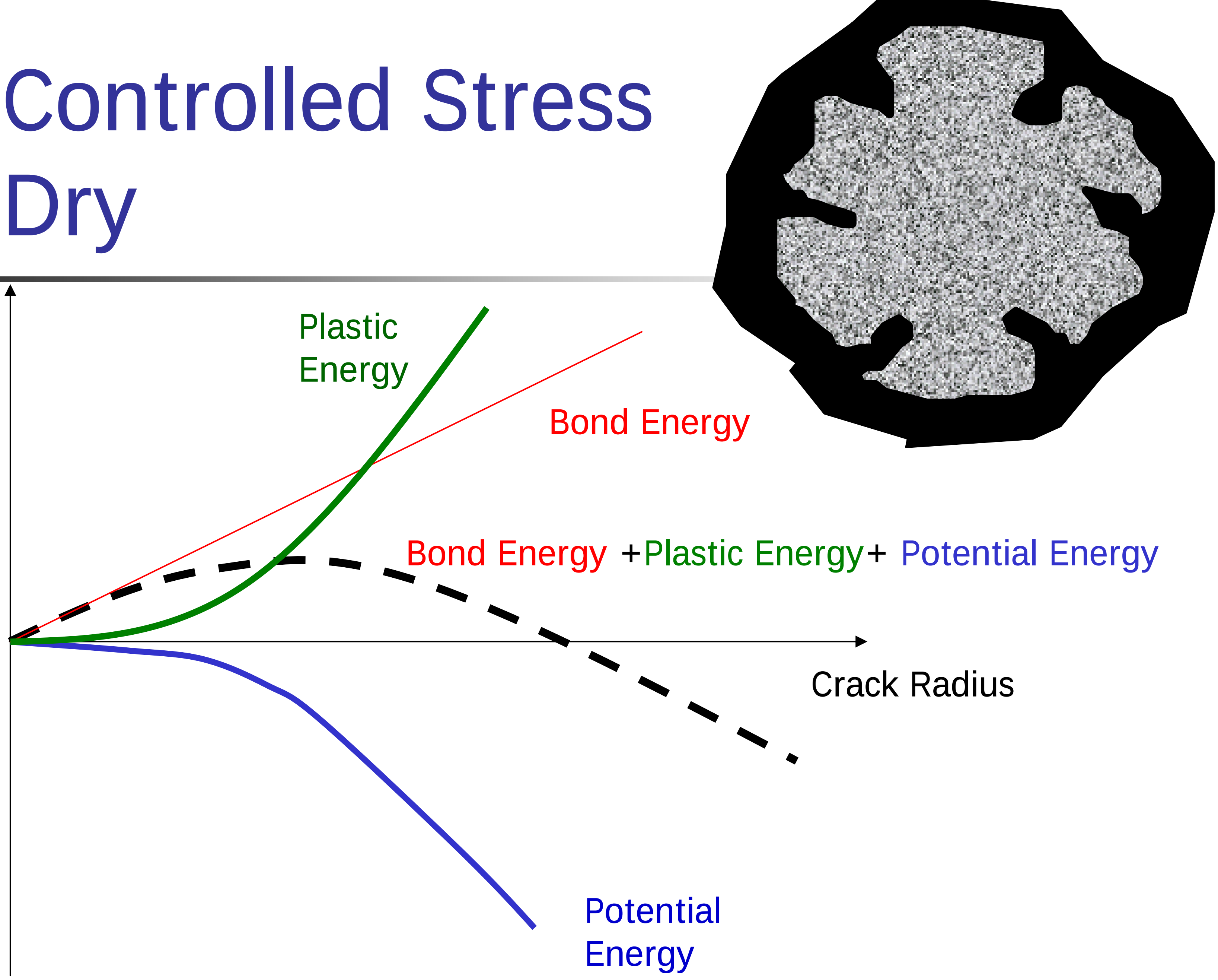
m = Number of cracks

$$\bar{r}(N) = \left(\frac{2n+1}{n+1}\right)^{\frac{n+1}{2n+1}} \left(\frac{A}{(4\pi m)^n}\right)^{\frac{1}{2n+1}} \left(\int_{N=0}^{N_f} \left(\frac{\partial W_R}{\partial N}\right)^{\frac{n}{n+1}} dN\right)^{\frac{n+1}{2n+1}}$$

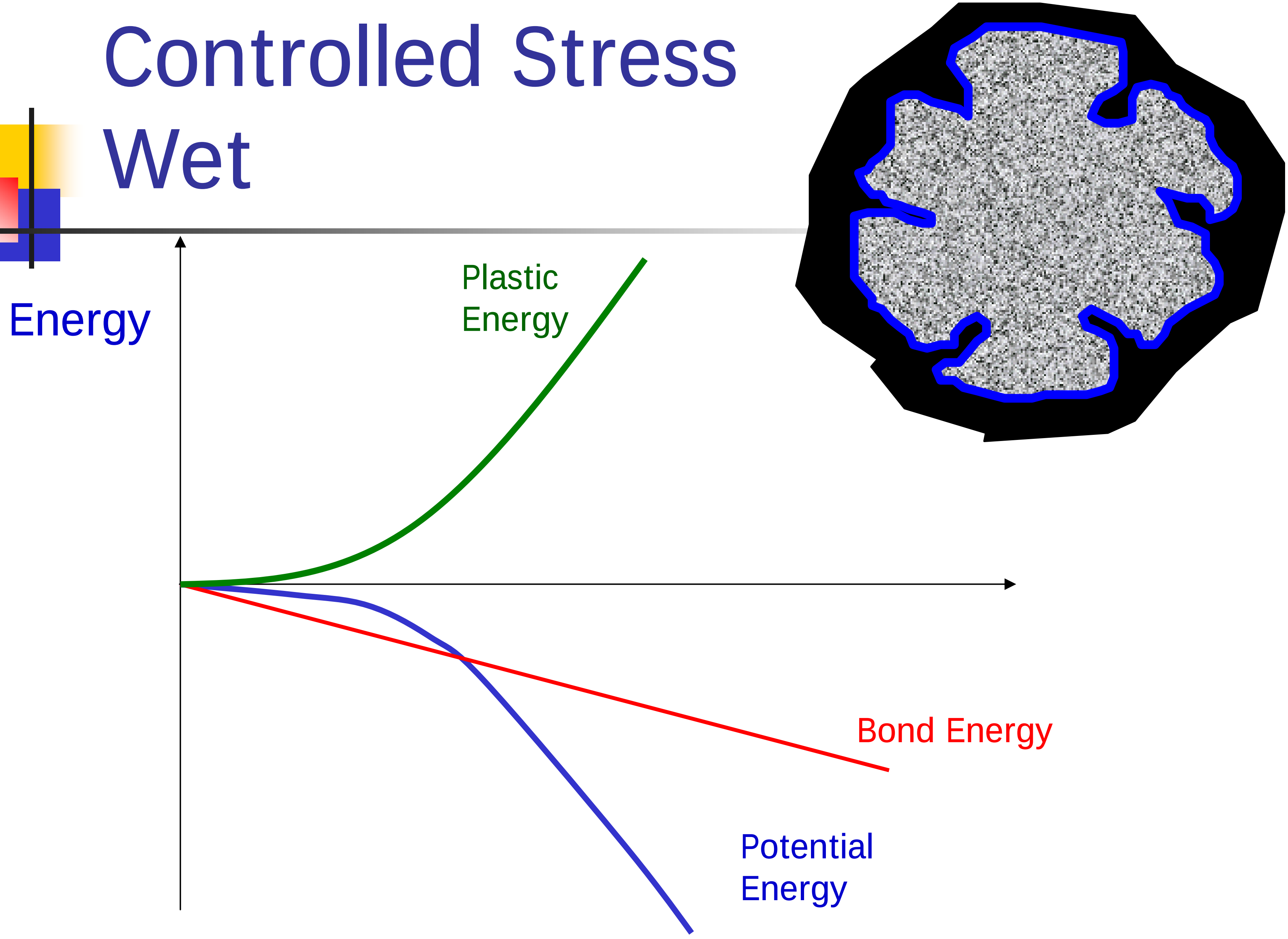
Controlled Strain Dry



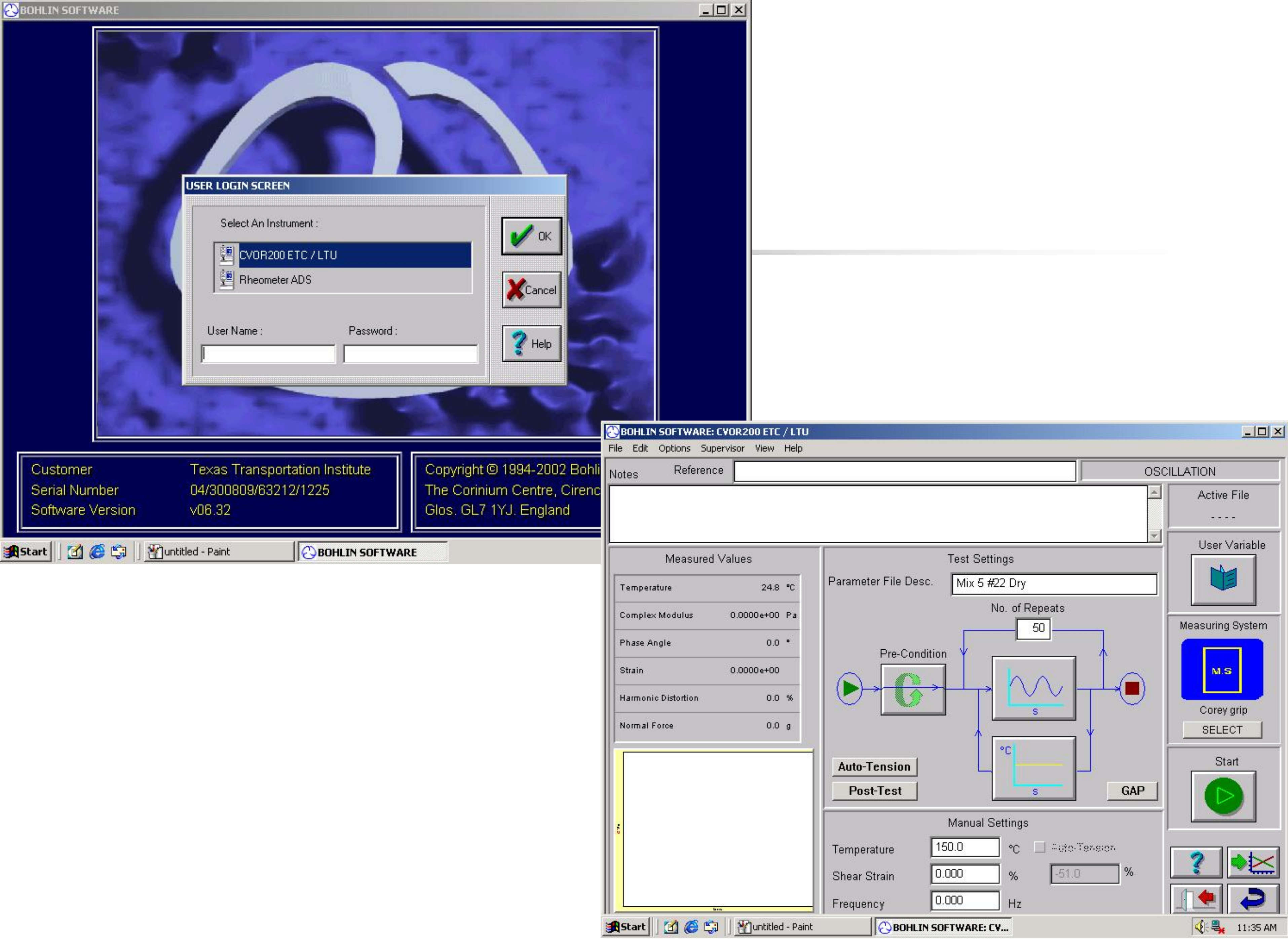
Controlled Stress Dry



Controlled Stress Wet




G-VOR



Select An Instrument :

- CVOR200 ETC / LTU
- Rheometer ADS

User Name :
Password :

Customer	Texas Transportation Institute
Serial Number	04/300809/63212/1225
Software Version	v06.32

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The Corinium Centre, Cirenc
Glos. GL7 1YJ. England

Measured Values

Temperature	24.8	°C
Complex Modulus	0.0000e+00	Pa
Phase Angle	0.0	°
Strain	0.0000e+00	
Harmonic Distortion	0.0	%
Normal Force	0.0	g

Test Settings

Parameter File Desc. Mix 5 #22 Dry

No. of Repeats 50

Measuring System: Corey grip

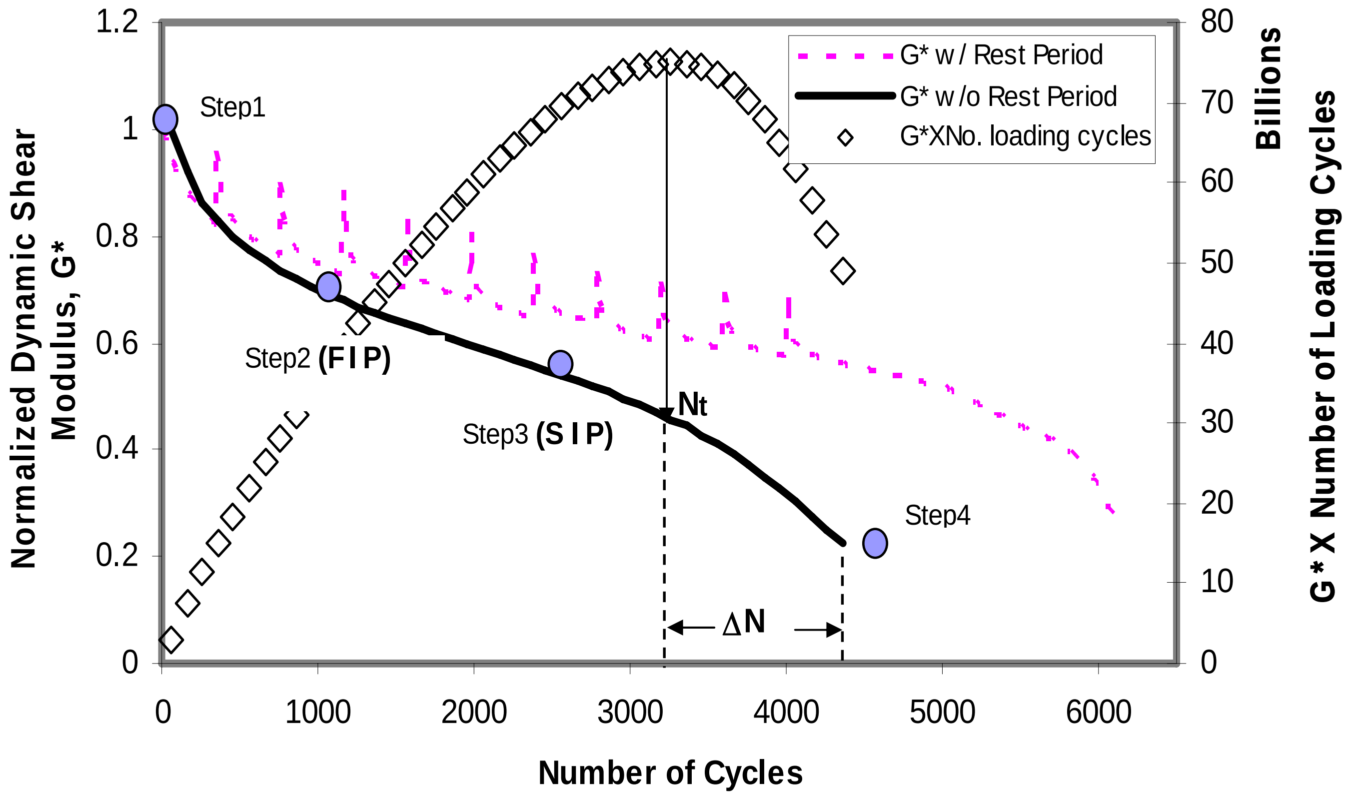
Manual Settings

Temperature	150.0	°C	☐ Auto-Tension		
Shear Strain	0.000	%		-51.0	%
Frequency	0.000	Hz			

Specimen Preparation

Analysis Parameters

- Failure Point



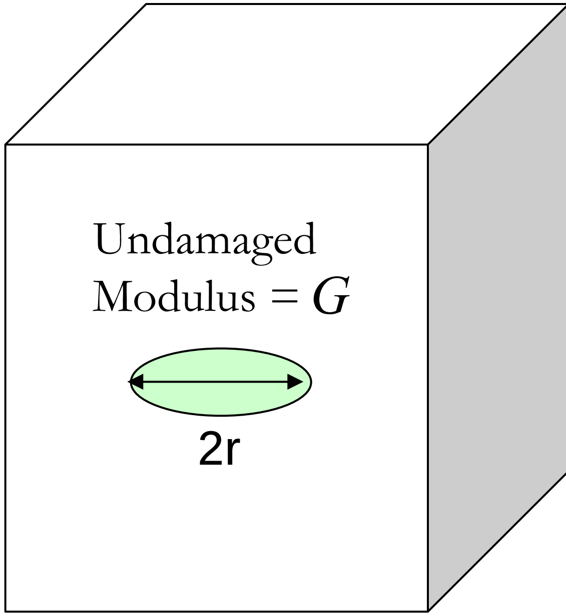
Analysis Parameters

- Dissipated Pseudo Strain Energy per Unit Volume of Stressed Material (W_R)
- Volume of Stressed Material:

$$Cohesive: \frac{G'}{G} = \left[1 - 2\pi^2\left(\frac{m}{A}\right)\frac{\overline{r}^3}{\overline{t}}\right]$$

$$Adhesive: \frac{G'}{G} = \left[1 - \pi^2\left(\frac{m}{A}\right)\frac{\overline{r}^3}{\overline{t}}\left[\left(1 + \frac{G_f}{G_s}\right)\right]\right]$$

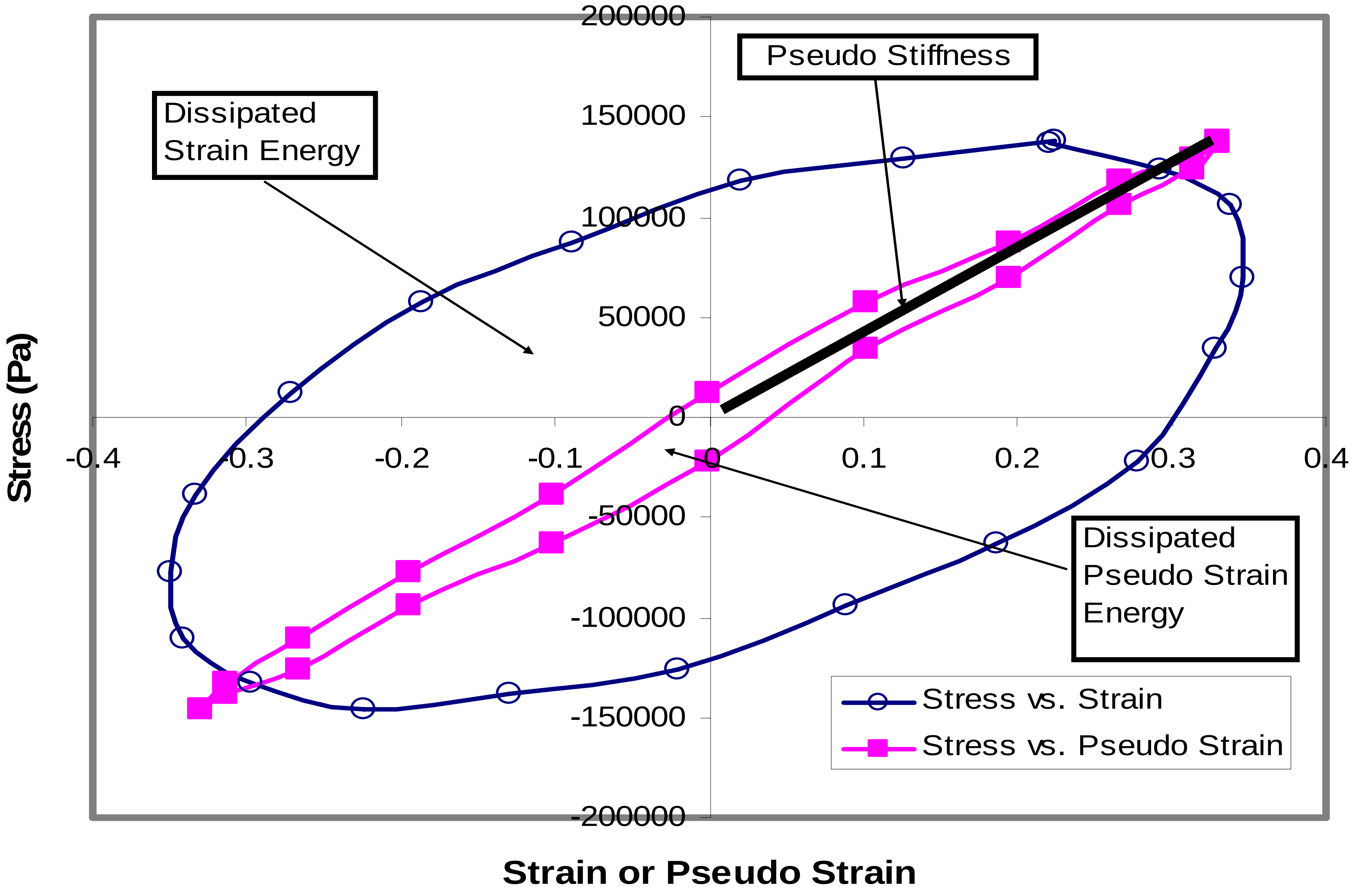
$$\tau_N = IC(S)\gamma_N^R$$



Undamaged Modulus = G

2r

Apparently Damaged Modulus = G'


Dissipated Strain Energy
Pseudo Stiffness
Dissipated Pseudo Strain Energy
200000
150000
100000
50000
0
-50000
-100000
-150000
-200000
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
Stress (Pa)
Strain or Pseudo Strain
Stress vs. Strain
Stress vs. Pseudo Strain

Definition of W_R

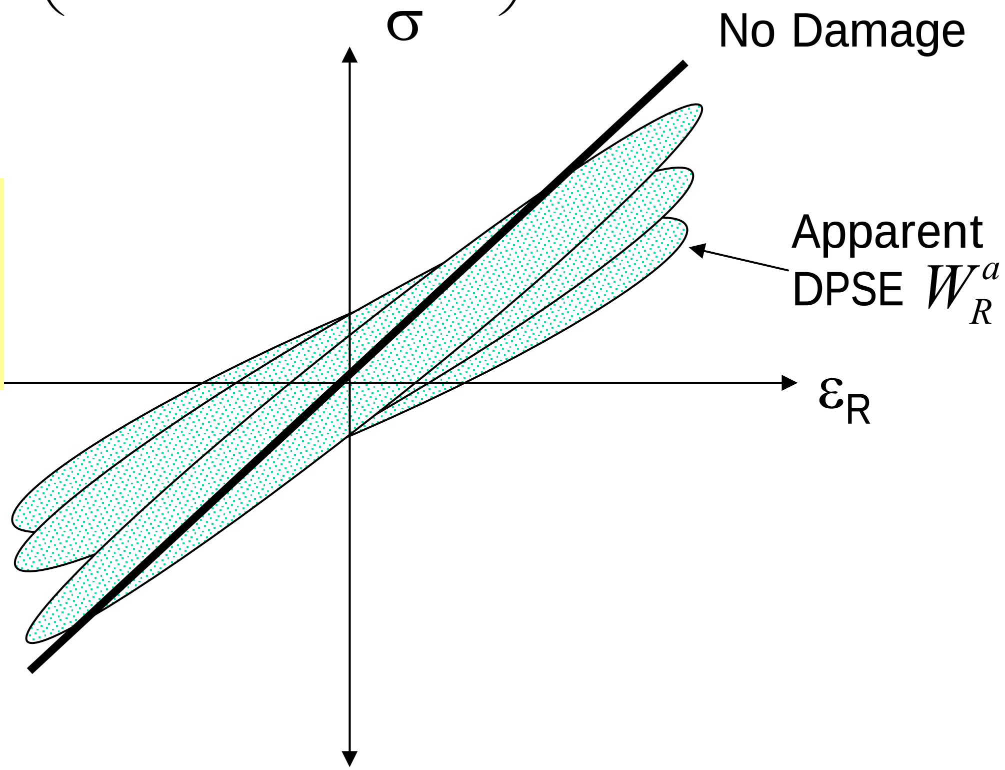
$$\bar{r}(N) = \left(\frac{2n+1}{n+1}\right)^{\frac{n+1}{2n+1}} \left(\frac{A}{(4\pi m)^{n}}\right)^{\frac{1}{2n+1}} \left(\int_{N=0}^{N_f} \left(\frac{\partial W_R}{\partial N}\right)^{\frac{n}{n+1}} dN\right)^{\frac{n+1}{2n+1}}$$

$$W_R = W_R^a/(G'/G)$$

Damage cannot be described by changes in linear viscoelastic properties

$$W_R \geq \pi S \varepsilon_{Ro}^2 \sin(\delta - \delta_{LVE})/(G'/G)$$

$$W_R \geq \pi I \varepsilon_{Ro}^2 \sin(\delta - \delta_{LVE})$$



Dissipated Pseudo Strain Energy (W_R)

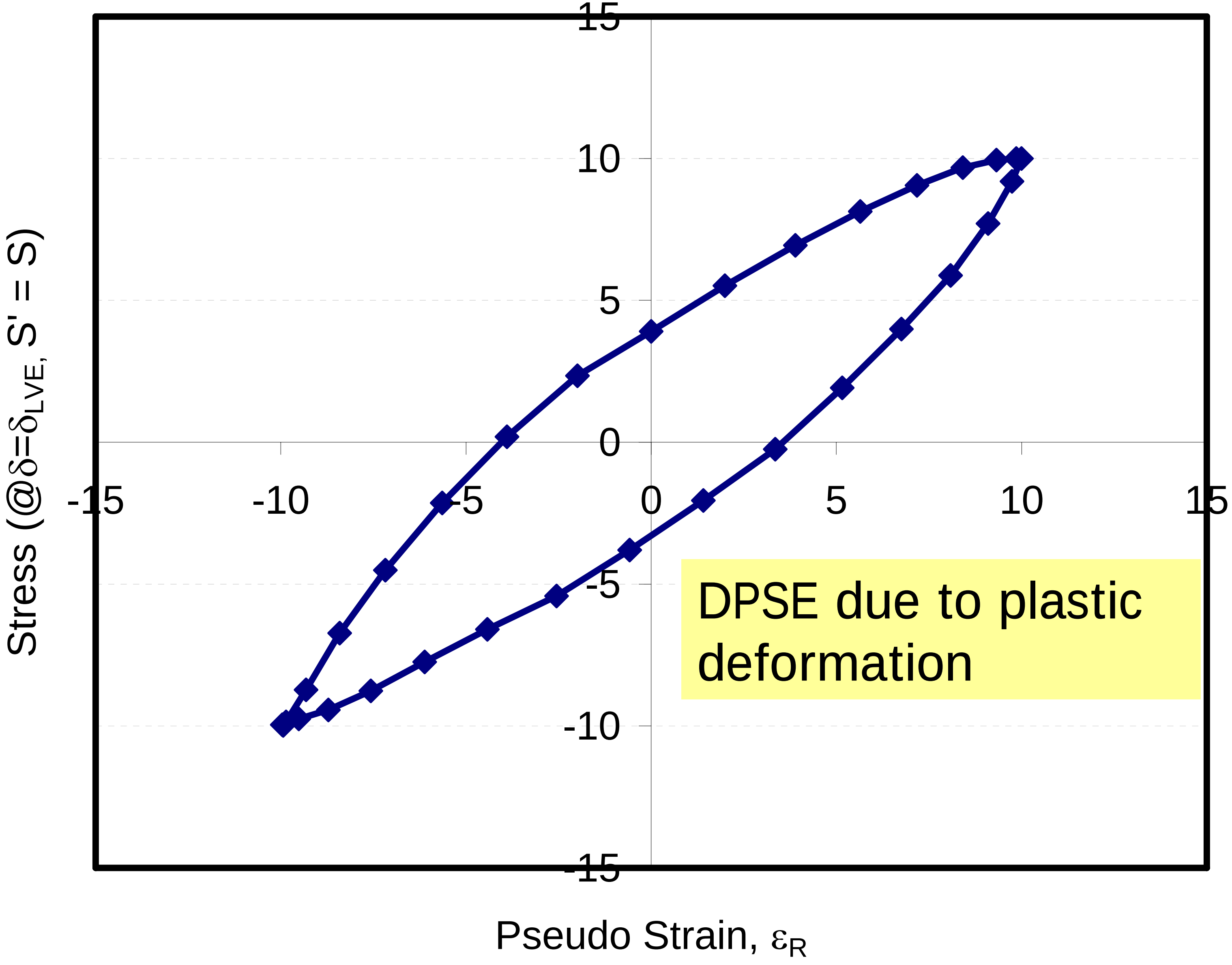
- Plastic deformation in each dynamic cycle and cracking causes “apparent” lag between stress and pseudo strain.
- The apparent lag angle (δ) is an average value calculated at the peaks of stress and strain functions.
- Lag angle is not the same throughout the cycle.
- The area in the hysteresis loop is due to cracking (fracture) (W_{RC}) and flow (plastic deformation) (W_{Rf}) .

Dissipated Pseudo Strain Energy (W_R)

- Assumption: Cracking causes uniform change in apparent lag between stress and pseudo strain.
- $W_R = W_{Rc} + W_{Rf}$

$$W_{Rc} = \pi S \varepsilon_{Ro}^2 \sin(\delta - \delta_{LVE}) / (G' / G)$$

- $W_{Rf} = W_R - W_{Rc}$



DPSE due to plastic deformation

W_R vs. Ln N

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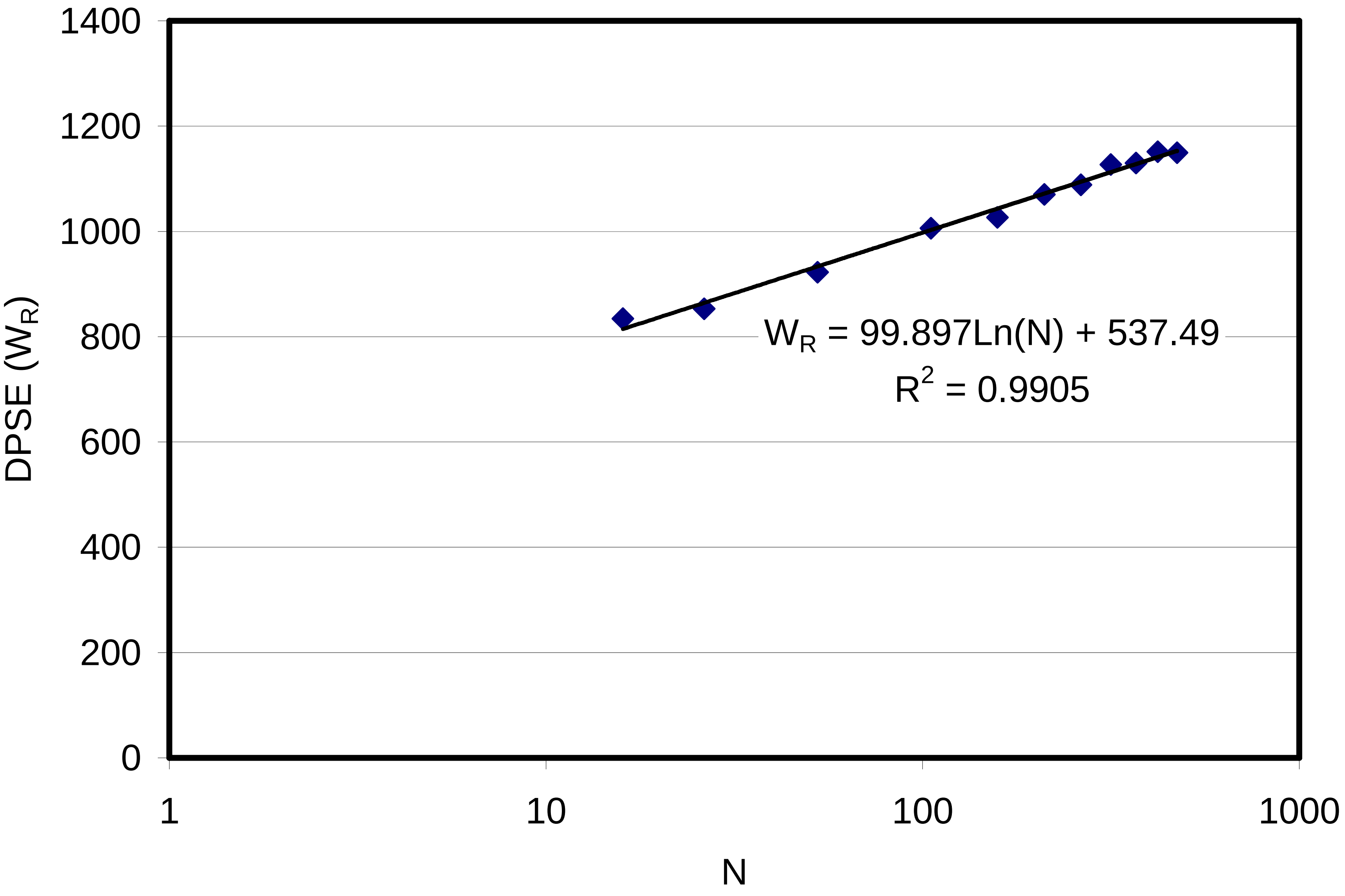


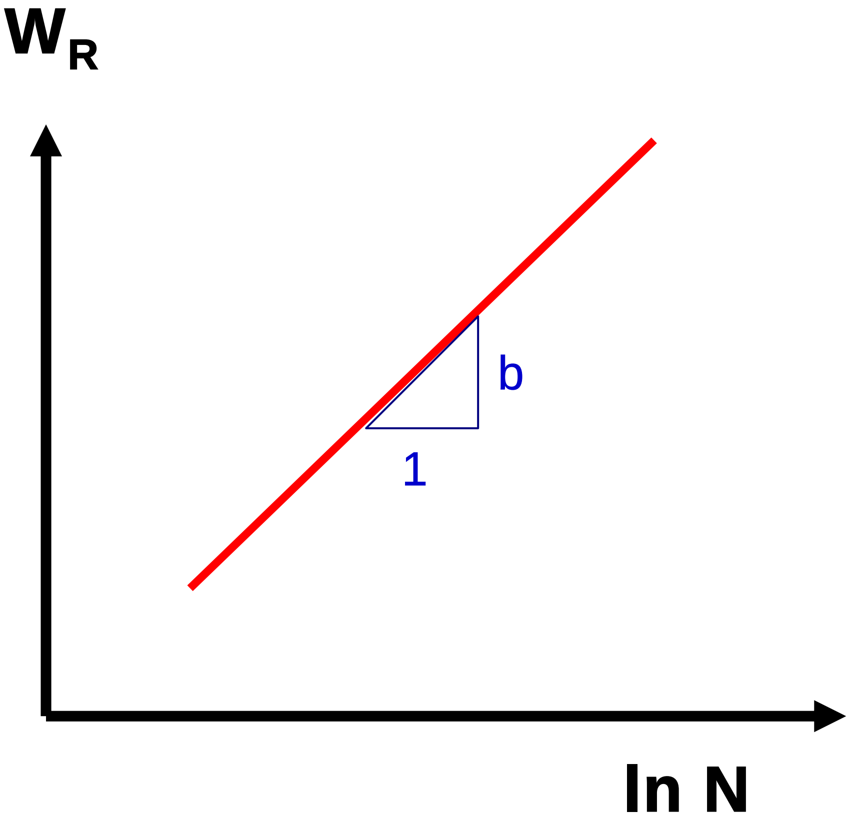
1400
1200
1000
800
600
400
200
0
DPSE (W_R)
1
10
100
1000
N
W_R = 99.897Ln(N) + 537.49
R^2 = 0.9905

Mixes

Mix #	Source	Aggregate Source	Binder Grade	Observed Field Performance
1	TX FM 369	MM Granite	Koch PG 76-22	Pass Hamburg Test
2	TX FM 369	MM Granite	Valero PG 76-22	Fail Hamburg Test
3	TX IH 20	MM Quartzite	Wright PG 76-22	Good
4	TX IH 20	Mer. Sandstone	Wright PG 76-22	Good
5	TX IH 20	Hanson Gravel	Wright PG 76-22	Good
6	TX IH 30	Mer. Sandstone	Lion PG 76-22	Good
7	Ohio	Gravel, Limestone	Tri-State PG 64-22	Poor
8	Ohio	Gravel, Rap	Marathon/Ashland PG 64-28	Poor

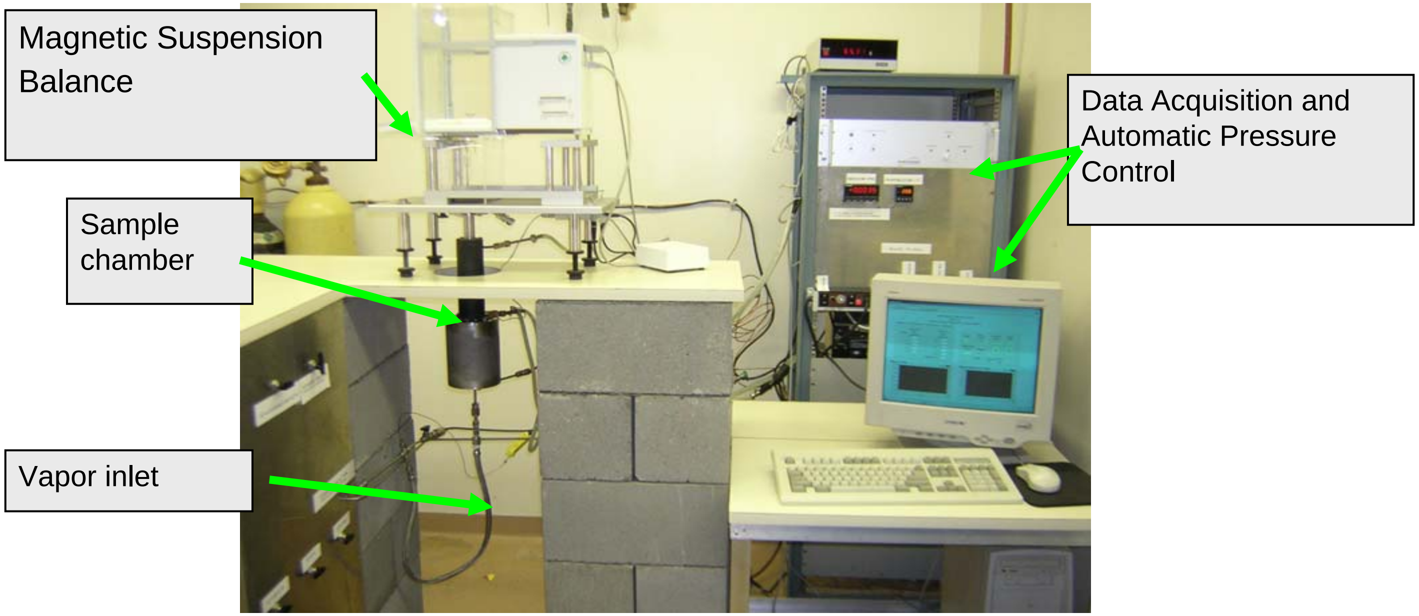
DMA Dissipated Pseudostrain Energy

Mix	Slope of DPSE (W_R) vs. Ln (N)	
	Dry	**Wet**
3	136.3	141.6
4	129.6	130.7
5	145.7	105.5
6	117.1	143.9
7	150.3	171.2
8	74.5	180.5



Aggregate Surface Energy

- Universal Sorption Device



Magnetic Suspension Balance

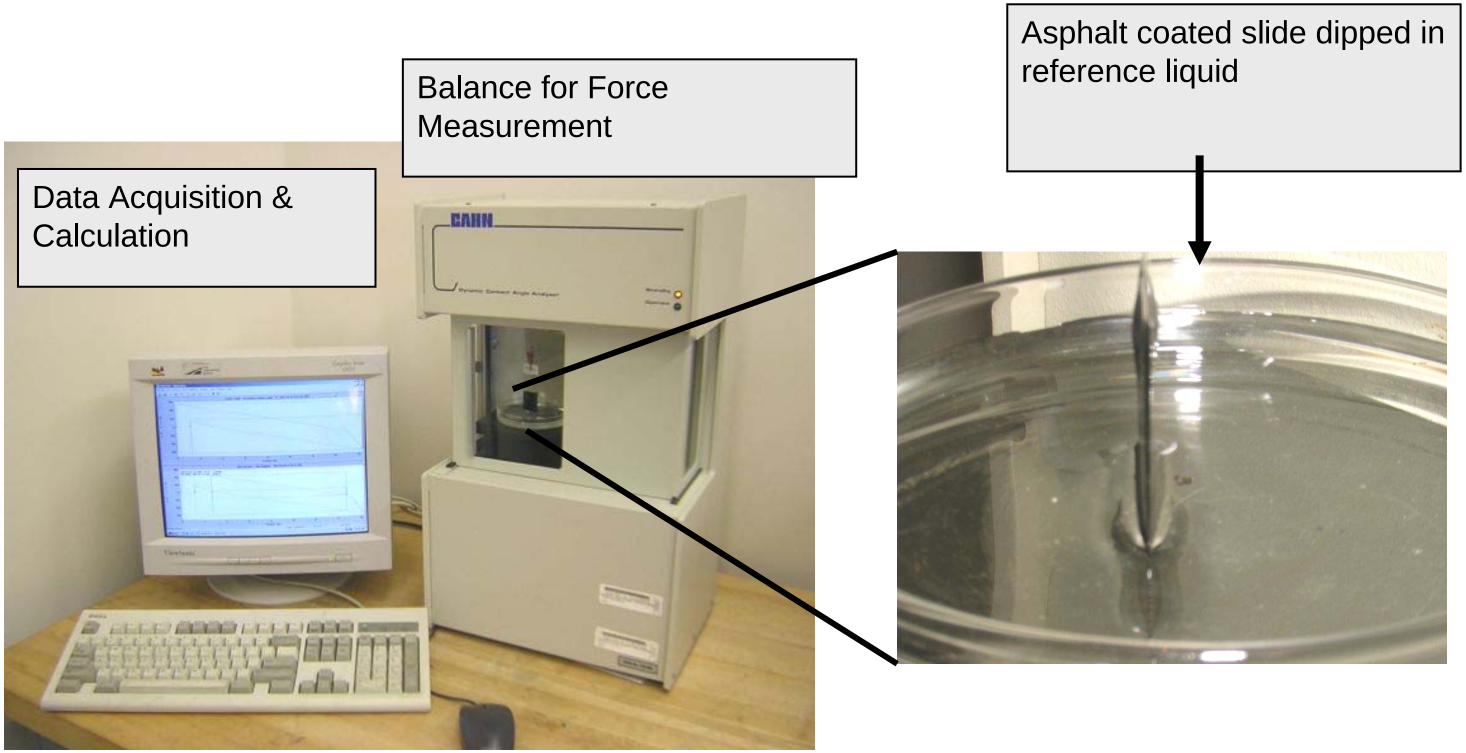
Sample chamber

Vapor inlet

Data Acquisition and Automatic Pressure Control

Binder Surface Energy

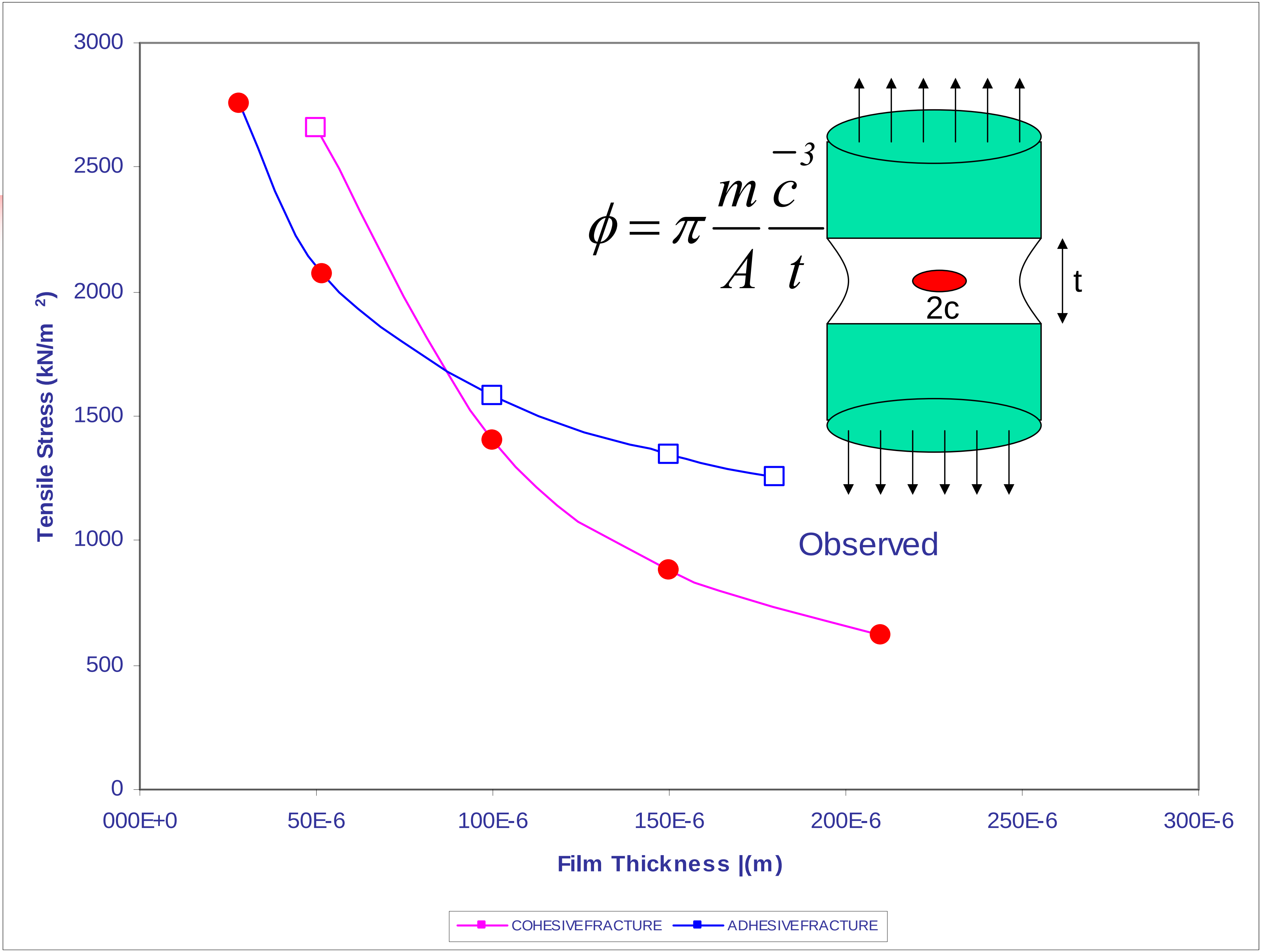
- Wilhelmy Plate



Data Acquisition & Calculation

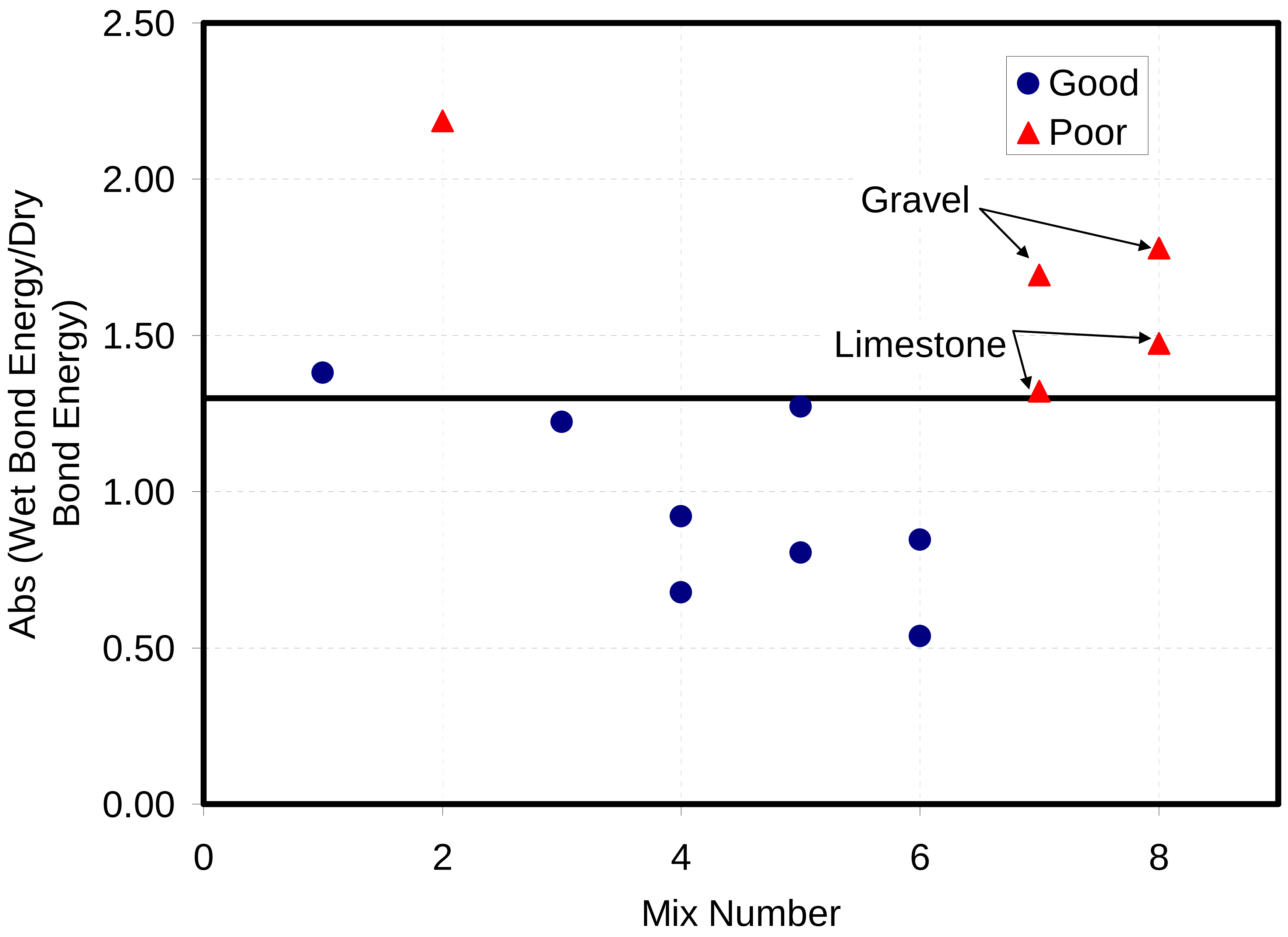
Balance for Force Measurement

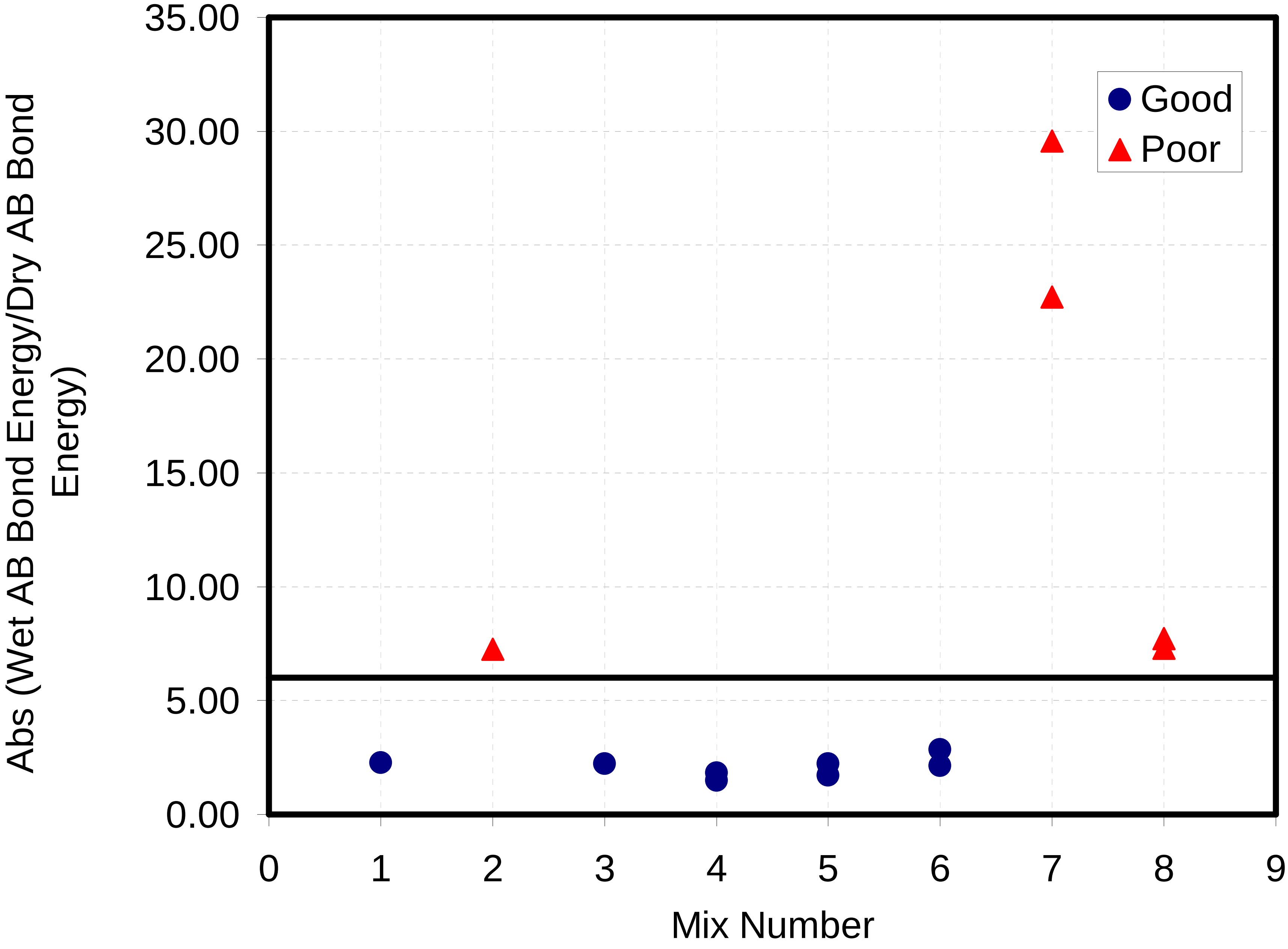
Asphalt coated slide dipped in reference liquid



$$\phi = \pi \frac{m}{A} \frac{c^{-3}}{t}$$

Mix Number	Aggregate Type	**ΔG^a Dry**	**$\Delta G^a{}_{AB}$ Dry**	**ΔG^a Wet**	**$\Delta G^a{}_{AB}$ Wet**	**Abs (ΔG^a Wet/ΔG^a Dry)**	**Abs ($\Delta G^a{}_{AB}$ Wet/$DG^a{}_{AB}$ Dry)**
1	Granite	140.61	83.51	-193.99	-189.08	1.38	2.26
2	Granite	104.72	31.75	-228.93	-230.02	2.19	7.24
3	Quartzite	114	59.62	-139.43	-132.01	1.22	2.21
4	Light Sandstone	91.57	36.48	-61.99	-54.34	0.68	1.49
4	Dark Sandstone	103.38	47.63	-95.25	-87.36	0.92	1.83
5	Gravel	93.36	39.59	-75.2	-68	0.81	1.72
5	TXI Limestone	118.87	64.92	-151.14	-143.87	1.27	2.22
6	Light Sandstone	99.61	25.1	-53.49	-53.78	0.54	2.14
6	Dark Sandstone	107.31	31.9	-90.86	-91.16	0.85	2.86
7	Limestone	87.49	4.07	-115.58	-120.34	1.32	29.57
7	Gravel	94.56	7.29	-160.22	-165.55	1.69	22.71
8	Limestone	81.27	15.27	-119.82	-117.84	1.47	7.72
8	Gravel	90.92	21.89	-161.87	-159.65	1.78	7.29


Abs (Wet Bond Energy/Dry Bond Energy)
2.50
2.00
1.50
1.00
0.50
0.00
0
2
4
6
8
Mix Number
● Good
▲ Poor
Gravel
Limestone



Abs (Wet AB Bond Energy/Dry AB Bond Energy)
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
0
1
2
3
4
5
6
7
8
9
Mix Number
● Good
▲ Poor

Total Bond Energy Ratio

> 0	< 0.65	Very Good
> 0.65	< 1.3	Good
> 1.3	< 1.95	Fair
	>1.95	Poor

	Quartzite	Sandstone 1	Granite	Gravel 1	Sandstone 2	Limestone 1	Limestone 2	Gravel 2
Wright PG 76-22	1.22	0.68	1.49	0.8	0.92	0.62	1.01	1.21
Lion PG 76-22	1.22	0.54	1.69	0.67	0.85	0.64	0.92	1.19
Valero PG 76-22	1.65	0.73	2.23	0.94	1.1	0.85	1.28	1.62
Koch PG 76-22	1.08	0.56	1.38	0.67	0.8	0.56	0.86	1.06
Tri-State PG 64-28	1.74	0.71	2.43	0.93	1.11	0.9	1.32	1.69
Marathon PG 64-22	1.81	0.88	2.27	1.11	1.24	0.91	1.47	1.78

AB Bond Energy Ratio

> 0	< 3	Very Good
> 3	< 6	Good
> 6	< 9	Fair
	>9	Poor

	Quartzite	Sandstone 1	Granite	Gravel 1	Sandstone 2	Limestone 1	Limestone 2	Gravel 2
Wright PG 76-22	2.21	1.49	2.37	1.72	1.83	1.08	2.01	2.22
Lion PG 76-22	3.44	2.14	3.93	2.47	2.86	1.74	2.95	3.43
Valero PG 76-22	8.16	5.84	7.63	7.1	6.45	3.7	8.24	8.29
Koch PG 76-22	2.01	1.28	2.27	1.47	1.67	1.01	1.75	2.01
Tri-State PG 64-28	21.66	17.78	15.67	25.79	15.38	8.39	29.59	22.73
Marathon PG 64-22	7.13	5.44	6.2	6.76	5.56	3.1	7.72	7.3

Comparison with DMA Results

Mix	Reported Performance	Slope of DSPE (W_R) vs. Ln (N)			Number of Cycles At Failure		
		Dry	Wet	Wet/Dry	Dry	Wet	Wet/Dry
3	Good	136.3	141.6	1.04	25,205	2,083	0.08
4	Good	129.6	130.7	1.01	16,349	14,671	0.90
5	Good	145.7	105.5	0.72	13,628	5,330	0.39
6	Good	117.1	143.9	1.23	13,541	5,603	0.41
7	Poor	150.3 (66.8)	171.2 (76.1)	1.14	3159 (6521)	803 (1633)	0.25
8	Poor	74.5 (33.1)	180.5 (80.2)	2.42	8767 (18,253)	2231 (4,590)	0.25

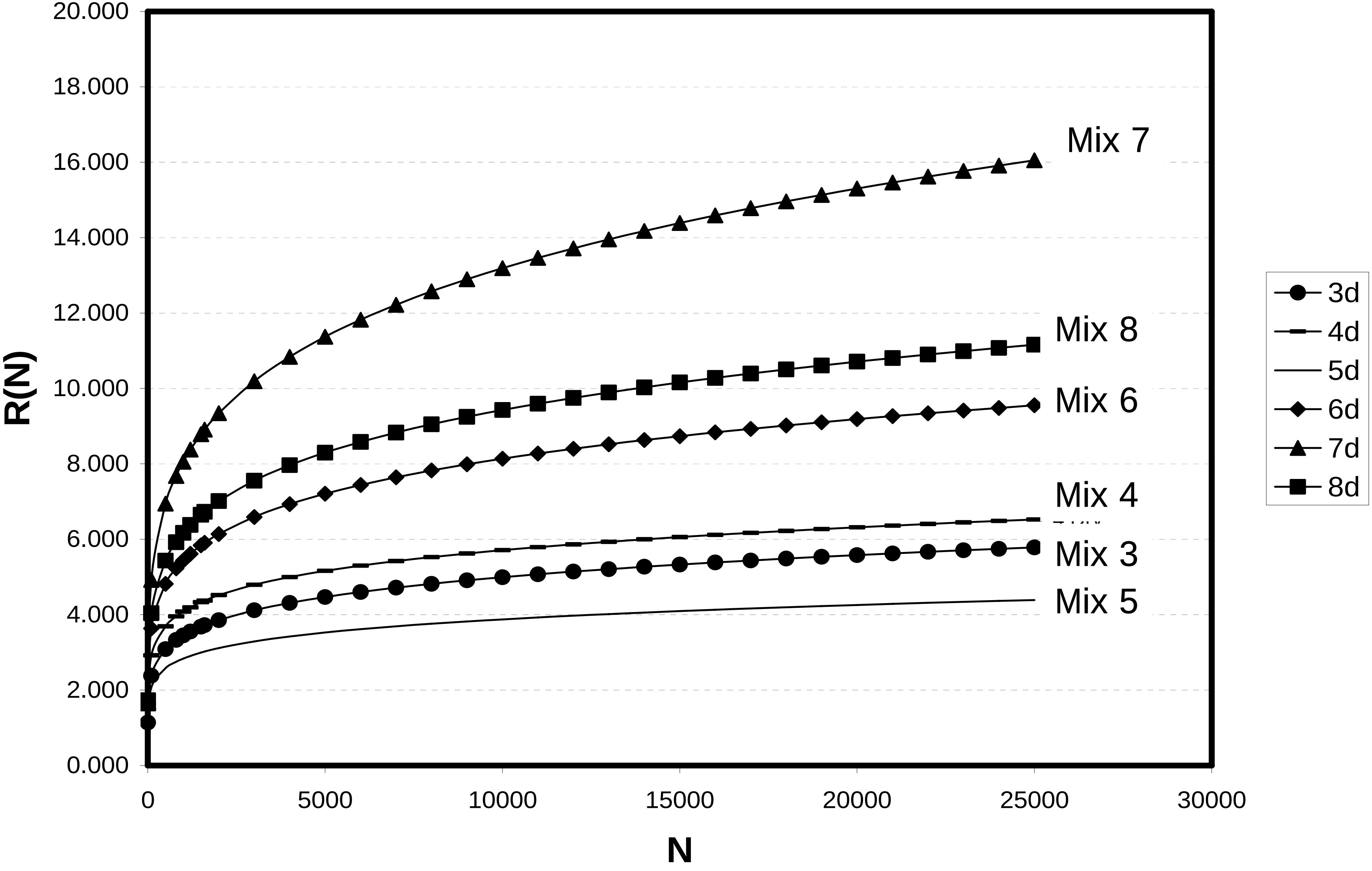
DMA
Mechanistic Approach

- Crack growth law used for both dry and wet conditions

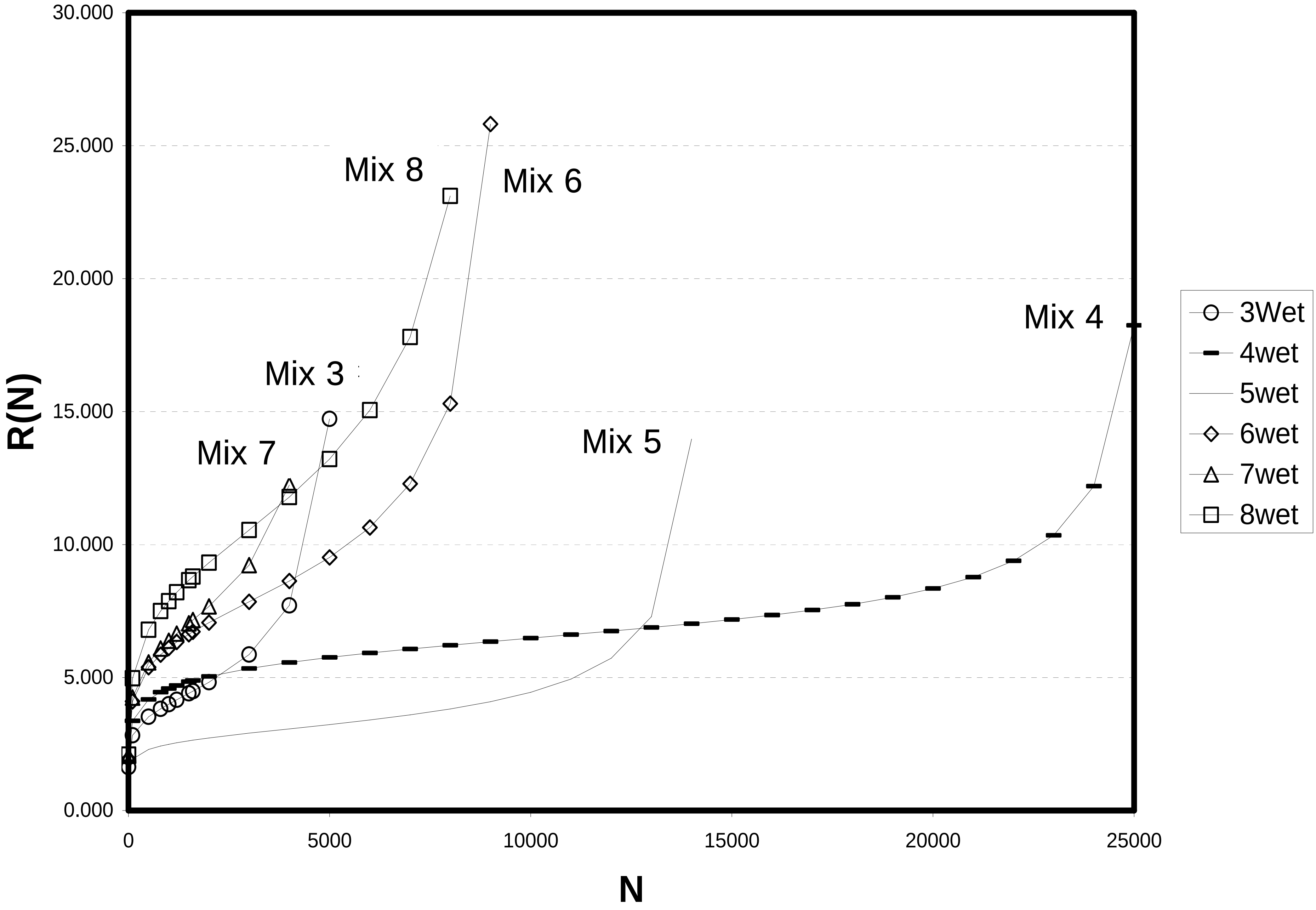
$$R = \frac{r(N)}{K^{\frac{1}{2n+1}}} = \left[(2n+1)^{n+1} \left(\frac{E_R b}{4\pi E_1 \Delta G_f} \right)^{n} N \right]^{\frac{1}{2n+1}}$$

- Partial wet surface energy value during wet test

$$\Delta G_{pw}^{a} = \frac{\left(\frac{G'}{G}\right)_w}{\left(\frac{G'}{G}\right)_D} \Delta G_d^{a} = \Delta G_d^{a}(1-P) + \Delta G_w^{a} P$$



20.000
18.000
16.000
14.000
12.000
10.000
8.000
6.000
4.000
2.000
0.000
R(N)
0
5000
10000
15000
20000
25000
30000
N
Mix 7
Mix 8
Mix 6
Mix 4
Mix 3
Mix 5
● 3d
▬ 4d
— 5d
◆ 6d
▲ 7d
■ 8d


30.000
25.000
20.000
15.000
10.000
5.000
0.000
R(N)
0
5000
10000
15000
20000
25000
N
Mix 8
Mix 6
Mix 3
Mix 7
Mix 5
Mix 4
○ 3Wet
▬ 4wet
— 5wet
◇ 6wet
△ 7wet
□ 8wet

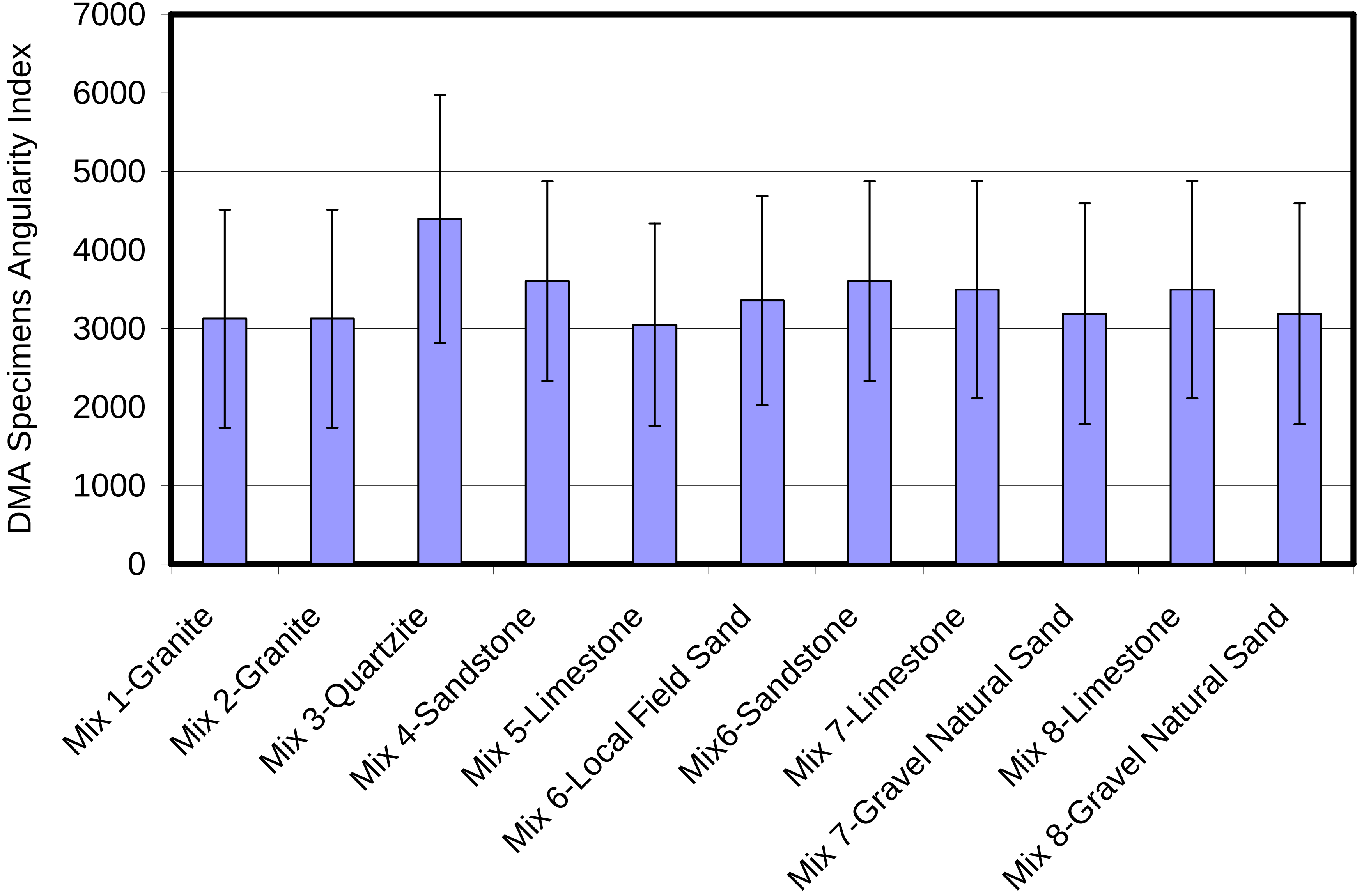
Angularity and Moisture Damage

- Angularity affects "A" and "n".
- As angularity increase, "n" increases and "A" tends to increase.

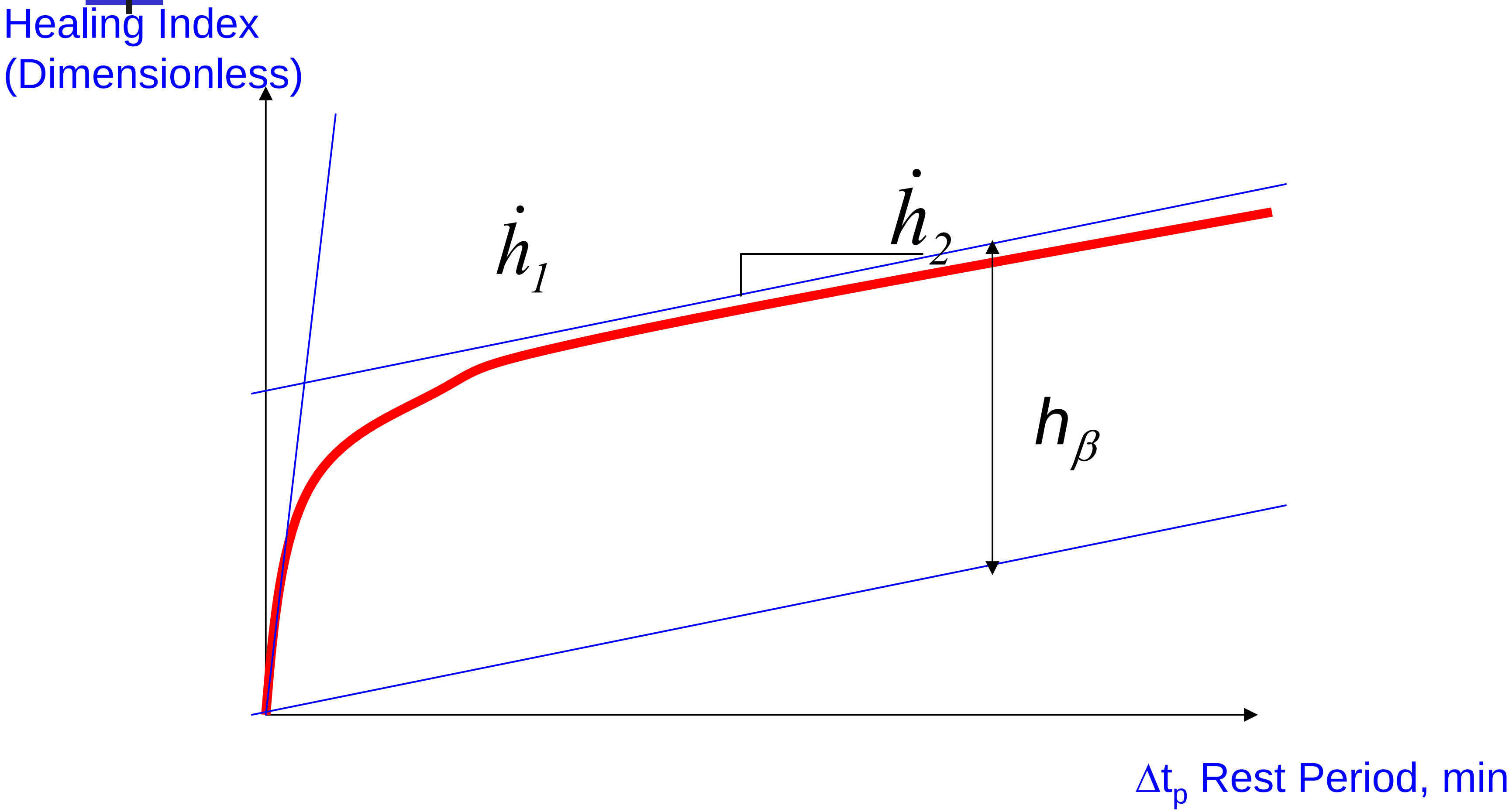
$$\frac{d\bar{r}}{dN} = A[J_R]^n$$

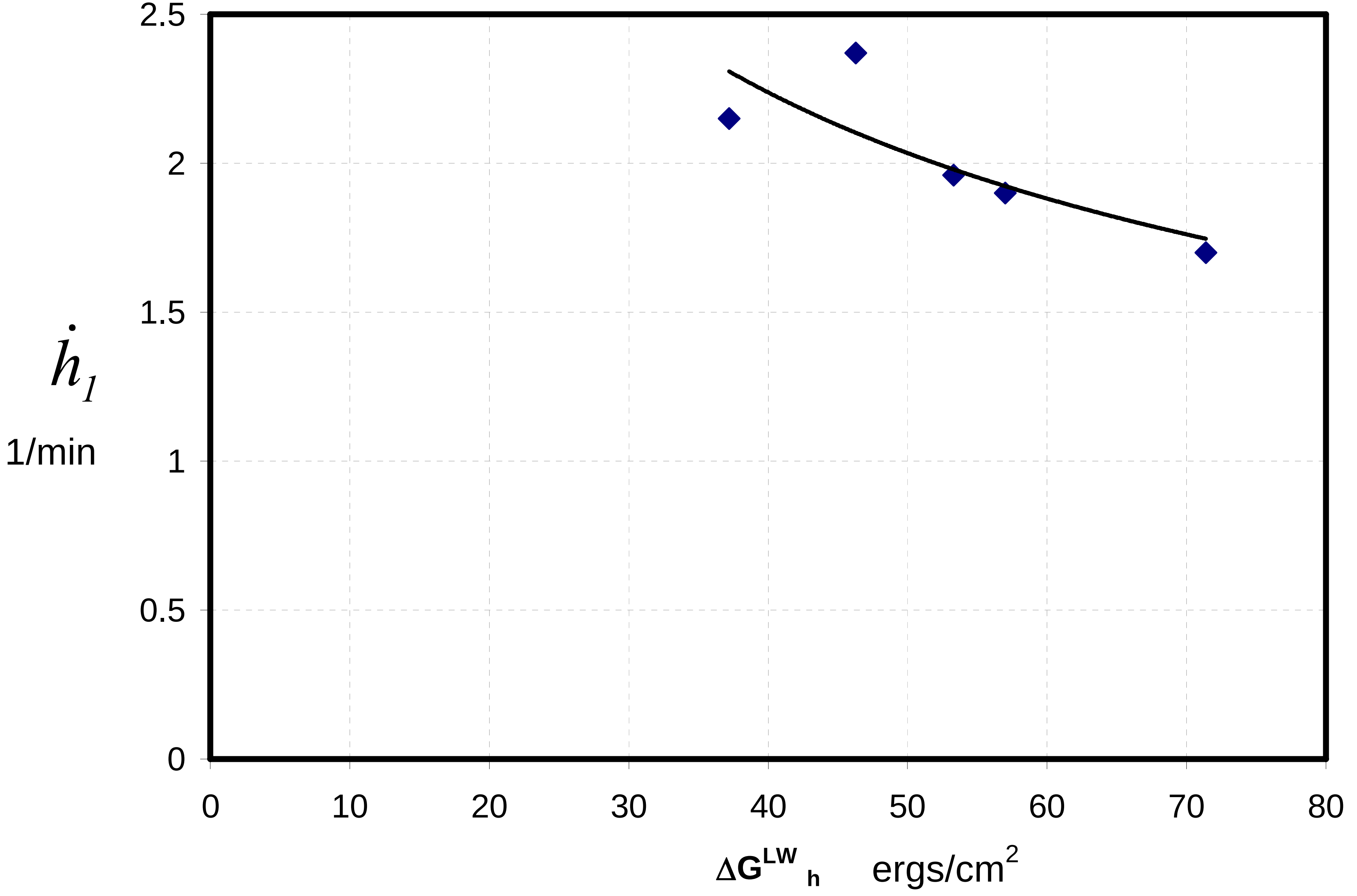
$$\bar{r}(N) = \left(\frac{2n+1}{n+1}\right)^{\frac{n+1}{2n+1}} \left(\frac{A}{(4\pi m)^n}\right)^{\frac{1}{2n+1}} \left(\int_{N=0}^{N_f} \left(\frac{\partial W_R}{\partial N}\right)^{\frac{n}{n+1}} dN\right)^{\frac{n+1}{2n+1}}$$

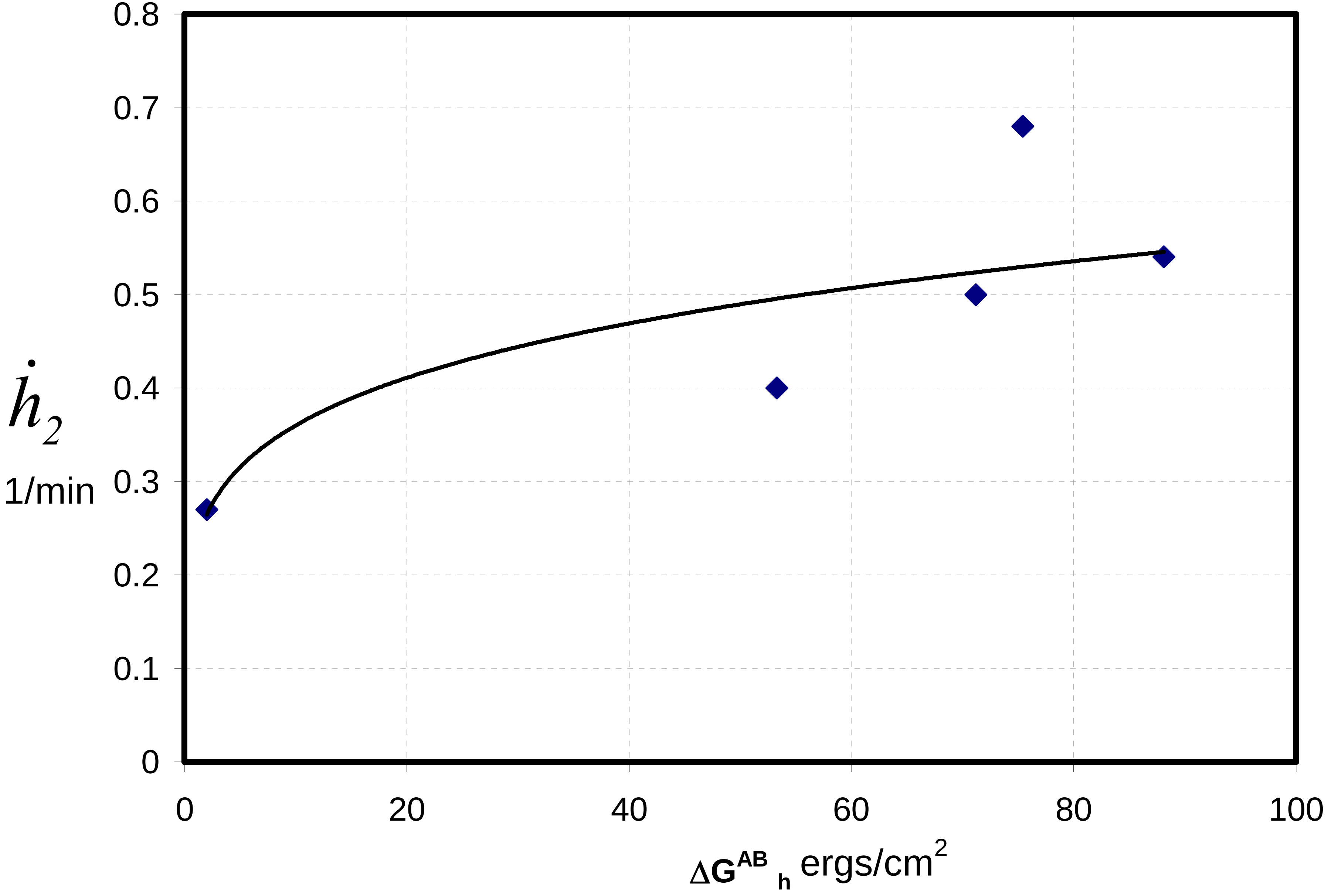
Aggregate Shape Characteristics Using AIMS


DMA Specimens Angularity Index
7000
6000
5000
4000
3000
2000
1000
0
Mix 1-Granite
Mix 2-Granite
Mix 3-Quartzite
Mix 4-Sandstone
Mix 5-Limestone
Mix 6-Local Field Sand
Mix6-Sandstone
Mix 7-Limestone
Mix 7-Gravel Natural Sand
Mix 8-Limestone
Mix 8-Gravel Natural Sand

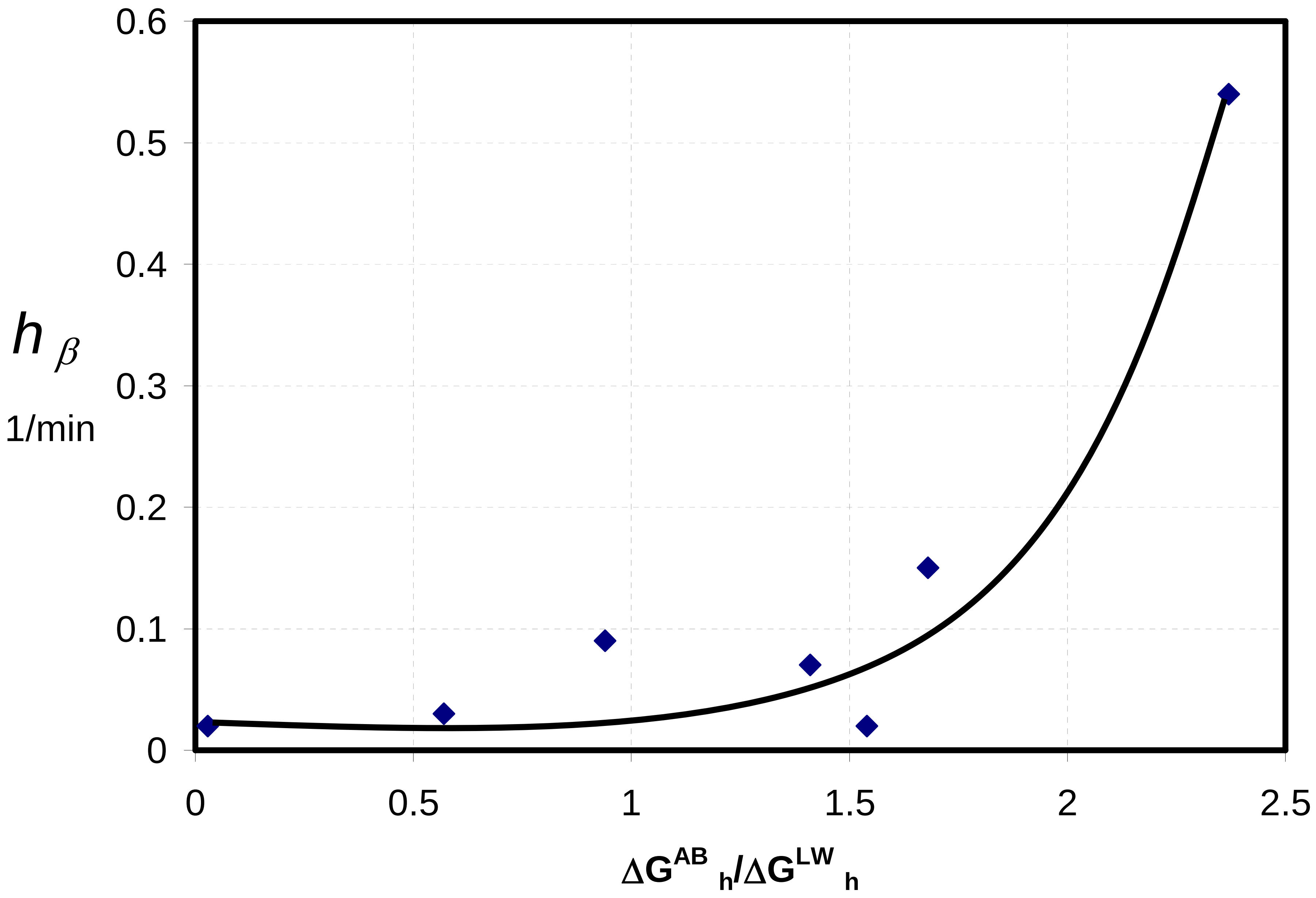
Healing


Healing Index (Dimensionless)
$\dot{h}_1$
$\dot{h}_2$
h_β
Δt_p Rest Period, min


2.5
2
1.5
1
0.5
0
$\dot{h}_1$
1/min
0
10
20
30
40
50
60
70
80
ΔG^{LW}_{h} ergs/cm^2



$\dot{h}_2$
1/min
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
ΔG^{AB}_{h} ergs/cm^2


h_β
1/min
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
$\Delta G^{AB}_h/\Delta G^{LW}_h$

Healing Functions

$$\dot{h}_1 = 4.89\left[\frac{E_1}{\Delta G_h^{LW}}\right]^{\frac{1}{m}}$$

$$\dot{h}_2 = 0.29 + 2.2\times 10^{20}\left[\frac{1}{E_1 \Delta G_h^{AB}}\right]^{\frac{1}{m}}$$

$$h_\beta = 0.011\left[\frac{\Delta G_h^{AB}}{\Delta G_h^{LW}}\right]^{\frac{1.62}{m}}$$

Concluding Remarks

- Prediction of adhesive fracture requires knowledge of:
  - Chemical (bond energy),
  - Mechanical (viscoelasticity, fracture plasticity),
  - Physical (aggregate angularity).
- The distinction between stress controlled and strain controlled testing is principally in the rate of energy dissipation in crack growth.

Concluding Remarks

- Water on the interface between aggregate and asphalt greatly reduces the energy that must be dissipated for cracks to grow.
- Ratio of total and Acid-Base wet bond energy to dry bond energy is a reliable predictor of resistance to adhesive moisture damage.

Concluding Remarks

- $\left[\dfrac{\Delta G_h^{AB}}{\Delta G_h^{LW}}\right]$ is a reliable predictor of healing

potential.

- DMA testing gives very good information about mix resistance to adhesive moisture damage.
- Mixes with thick asphalt films will fracture and heal with cohesive properties of the mastic.